```
inj_imp_le  THM

⊢  (∃f:(ℕm→ℕk). Inj(ℕm; ℕk; f)) ⇒ m≤k
| by Induction on m, with trivial base case 0≤k
|
1. m : ℤ
2. 0<m
3. ∀k':ℕ. (∃f':(ℕ(m-1)→ℕk'). Inj(ℕ(m-1); ℕk'; f')) ⇒ m-1≤k'
4. k : ℕ
5. ∃f:(ℕm→ℕk). Inj(ℕm; ℕk; f)
⊢  m≤k  by Analyze5
|
5. f : ℕm→ℕk
6. Inj(ℕm; ℕk; f)
⊢  m≤k  by m-1≤k-1  Asserted THEN k-1 ∈ ℕ  Asserted
|
|\
| .....assertion .....
| ⊢  k-1 ∈ ℕ  by f(0) ∈ ℕk  Asserted
|
.\
. 7. k-1 ∈ ℕ
. ⊢  m-1≤k-1  by BackThru: Hyp:3
. |
. ⊢  ∃f':(ℕ(m-1)→ℕ(k-1)). Inj(ℕ(m-1); ℕ(k-1); f')
. | by Witness: Replace value k-1 by f(m-1) in f
. |
. ⊢  Inj(ℕ(m-1); ℕ(k-1); Replace value k-1 by f(m-1) in f)
.   by BackThru:
.      Thm*  Inj(ℕ(m+1); ℕ(k+1); f)
.            ⇒
.            Inj(ℕm; ℕk; Replace value k by f(m) in f)
.      {delete_fenum_value_is_inj}
END
```