```
inv_funs_iff_inv_funs_2  THM

⊢  InvFuns(A; B; f; g) ⟺ InvFuns(A;B;f;g)
| by Unfolds [`inv_funs_2`;`inv_funs`] 0
|
⊢  ∀A,B:Type, f:(A→B), g:(B→A).
|  g o f = Id & f o g = Id
|  ⟺
|  (∀x:A. g(f(x)) = x) & (∀y:B. f(g(y)) = y)
| by ∀A,B:Type, f:(A→B), g:(B→A). g o f = Id ⟺ (∀x:A. g(f(x)) = x)
|    Asserted
|
|\
| .....assertion .....
| ⊢  ∀A,B:Type, f:(A→B), g:(B→A). g o f = Id ⟺ (∀x:A. g(f(x)) = x)
| | by UnivCD
| |
| 1. A : Type
| 2. B : Type
| 3. f : A→B
| 4. g : B→A
| ⊢  g o f = Id ⟺ (∀x:A. g(f(x)) = x)  by Auto
| |
| |\
| | 5. g o f = Id
| | 6. x : A
| | ⊢  g(f(x)) = x
| |   by Compute ⌜g(f(x)) = x ∈ A⌝ *⇒ (g o f)(x) = Id(x) THEN Analyze
| |
| .\
| . 5. ∀x:A. g(f(x)) = x
| . ⊢  g o f = Id  by Ext
| . |
| . 6. x : A
| . ⊢  (g o f)(x) = Id(x)
| .   by Compute ⌜(g o f)(x) = Id(x)⌝ *⇒ g(f(x)) = x ∈ A
| .      THEN
| .      BackThru: Hyp:5
|
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. 1. ∀A,B:Type, f:(A→B), g:(B→A). g o f = Id ⟺ (∀x:A. g(f(x)) = x)
. ⊢  ∀A,B:Type, f:(A→B), g:(B→A).
.    g o f = Id & f o g = Id
.    ⟺
.    (∀x:A. g(f(x)) = x) & (∀y:B. f(g(y)) = y)
.   by Auto THEN BackThru: Hyp:1
END
```