```
inv_pair_functionality_wrt_one_one_corr  THM

⊢  (A ∼ A') ⇒ (B ∼ B') ⇒ ((A←→B) ∼ (A'←→B'))  by Auto
|
1. A : Type
2. A' : Type
3. B : Type
4. B' : Type
5. A ∼ A'
6. B ∼ B'
⊢  (A←→B) ∼ (A'←→B')
| by Analyze-2 THEN Analyze-2 THEN Analyze-2
|    THEN
|    New:f' Analyze-1 THEN New:g' Analyze-1 THEN Analyze-1
|    THEN
|    Def
|
5. f : A→A'
6. g : A'→A
7. ∀x:A. g(f(x)) = x
8. ∀y:A'. f(g(y)) = y
9. f' : B→B'
10. g' : B'→B
11. ∀x:B. g'(f'(x)) = x
12. ∀y:B'. f'(g'(y)) = y
⊢  ∃f:((A←→B)→A'←→B'), g:((A'←→B')→A←→B). InvFuns(A←→B;A'←→B';f;g)
| by (λde.de/d,e. <f' o d o g,f o e o g'>) ∈ (A←→B)→A'←→B' |
|    (λde.de/d,e. <g' o d o f,g o e o f'>) ∈ (A'←→B')→A←→B
|    Asserted
|
|\
| .....assertion .....
| ⊢  (λde.de/d,e. <f' o d o g,f o e o g'>) ∈ (A←→B)→A'←→B'
| | by (Analyze THEN Analyze-1)
| |    THEN
| |    New:[d | e] Analyze-2 THEN Reduce 0 THEN Reduce -1
| |    THEN
| |    Analyze THEN Reduce Concl
| |    THEN
| |    Analyze-1
| |
| 13. d : A→B
| 14. e : B→A
| 15. ∀x:A. e(d(x)) = x
| 16. ∀y:B. d(e(y)) = y
| ⊢  InvFuns(A';B';f' o d o g;f o e o g')
|   by Analyze THEN Reduce 0
|      THEN
|      3 Times
|      Rewrite by Hyp:16 | Hyp:15 | Hyp:12 | Hyp:11 | Hyp:8 | Hyp:7
|
|\
| .....assertion .....
| 13. (λde.de/d,e. <f' o d o g,f o e o g'>) ∈ (A←→B)→A'←→B'
| ⊢  (λde.de/d,e. <g' o d o f,g o e o f'>) ∈ (A'←→B')→A←→B
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| | by Thin -1
| |
| 12. ∀y:B'. f'(g'(y)) = y
| ⊢  (λde.de/d,e. <g' o d o f,g o e o f'>) ∈ (A'←→B')→A←→B
|   by (Analyze THEN Analyze-1)
|      THEN
|      New:[d | e] Analyze-2 THEN Reduce 0 THEN Reduce -1
|      THEN
|      Analyze THEN Reduce Concl
|      THEN
|      Analyze-1
|      THEN
|      Analyze THEN Reduce 0
|      THEN
|      3 Times
|      Rewrite by Hyp:16 | Hyp:15 | Hyp:12 | Hyp:11 | Hyp:8 | Hyp:7
|
.\
. 13. (λde.de/d,e. <f' o d o g,f o e o g'>) ∈ (A←→B)→A'←→B'
. 14. (λde.de/d,e. <g' o d o f,g o e o f'>) ∈ (A'←→B')→A←→B
. ⊢  ∃f:((A←→B)→A'←→B'), g:((A'←→B')→A←→B). InvFuns(A←→B;A'←→B';f;g)
. | by Witness:
. |      λde.de/d,e. <f' o d o g,f o e o g'> |
. |      λde.de/d,e. <g' o d o f,g o e o f'>
. |    THEN
. |    Analyze THEN Analyze15 THEN New:[d | e] Analyze15
. |
. |\
. | 15. d : A→B
. | 16. e : B→A
. | 17. <d,e>/f,g. InvFuns(A;B;f;g)
. | ⊢  (λde.de/d,e. <g' o d o f,g o e o f'>)
. | |  ((λde.de/d,e. <f' o d o g,f o e o g'>)(<d,e>))
. | |  =
. | |  <d,e>
. | |  ∈ A←→B  by Reduce Hyp:-1 THEN Analyze-1 THEN Reduce Concl
. | |
. | 17. ∀x:A. e(d(x)) = x
. | 18. ∀y:B. d(e(y)) = y
. | ⊢  <g' o f' o d o g o f,g o f o e o g' o f'> = <d,e> ∈ A←→B
. | | by (2 Times Analyze)
. | |    THEN
. | |    ...
. | |
. | .....set predicate .....
. | ⊢  <g' o f' o d o g o f,g o f o e o g' o f'>/f,g. InvFuns(A;B;f;g)
. |   by Reduce 0 THEN Analyze THEN Reduce 0
. |      THEN
. |      5 Times
. |      Rewrite by Hyp:18 | Hyp:17 | Hyp:12 | Hyp:11 | Hyp:8 | Hyp:7
. |
. .\
. . 15. d : A'→B'
. . 16. e : B'→A'
. . 17. <d,e>/f,g. InvFuns(A';B';f;g)
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. . ⊢  (λde.de/d,e. <f' o d o g,f o e o g'>)
. . |  ((λde.de/d,e. <g' o d o f,g o e o f'>)(<d,e>))
. . |  =
. . |  <d,e>
. . |  ∈ A'←→B'  by Reduce -1 THEN Reduce 0
. . |
. . 17. InvFuns(A';B';d;e)
. . ⊢  <f' o g' o d o f o g,f o g o e o f' o g'> = <d,e> ∈ A'←→B'
. .   by Reduce Hyp:-1 THEN Analyze-1 THEN Reduce Concl
. .      THEN
. .      ((2 Times Analyze)
. .       THEN
. .       (FunExtensionality
. .        THEN
. .        Reduce 0
. .        THEN
. .        2 Times
. .        Rewrite by
. .        Hyp:18 |
. .        Hyp:17 |
. .        Hyp:12 |
. .        Hyp:11 |
. .        Hyp:8 |
. .        Hyp:7))
. .      THEN
. .      Reduce 0 THEN Analyze THEN Reduce 0
. .      THEN
. .      5 Times
. .      Rewrite by Hyp:18 | Hyp:17 | Hyp:12 | Hyp:11 | Hyp:8 | Hyp:7
END
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