```
iter_perm_cycles_uniform  THM

⊢  ∀k:ℕ, f:(ℕk inj→ ℕk). ∃i:ℕ+. ∀u:ℕk. f{0i}(u) = u
| by SimilarTo
|    Thm*  ∀k:ℕ, f:(ℕk inj→ ℕk), u:ℕk. ∃i:ℕ+. f{0i}(u) = u
|    {compose_iter_inj_cycles} ...wℕ
|
1. k : ℕ
2. f : ℕk inj→ ℕk
3. ∀u:ℕk. ∃i:ℕ+. f{0i}(u) = u
⊢  ∃i:ℕ+. ∀u:ℕk. f{0i}(u) = u  by Analyze2
|
2. f : ℕk→ℕk
3. Inj(ℕk; ℕk; f)
4. ∀u:ℕk. ∃i:ℕ+. f{0i}(u) = u
⊢  ∃i:ℕ+. ∀u:ℕk. f{0i}(u) = u
| by FwdThru:
|    Thm*  ∀k:ℕ, Q:(ℕk→A→Prop).
|          (∀x:ℕk. ∃y:A. Q(x;y)) ⇒ (∃f:(ℕk→A). ∀x:ℕk. Q(x;f(x)))
|    {finite_choice}
|    on [ Hyp:-1 ]
|    THEN
|    New:h Analyze-1
|
5. h : ℕk→ℕ+
6. ∀u:ℕk. f{0h(u)}(u) = u
⊢  ∃i:ℕ+. ∀u:ℕk. f{0i}(u) = u
| by SimilarTo: ∃i:ℕ+. ∀u:ℕk. ∃m:ℕ. i = h(u)·m ∈ ℕ ...ℕ
|
|\
| .....assertion .....
| ⊢  ∃i:ℕ+. ∀u:ℕk. ∃m:ℕ. i = h(u)·m ∈ ℕ
| | by Witness: Π u:ℕk. h(u) ...ℕ
| |
| |\
| | 7. x : ℕ+
| | 8. y : ℕ+
| | ⊢  x·y ∈ ℕ+
| |   by BackThru: Thm*  ∀x,y:ℕ+. x·y ∈ ℕ+    {mul_nat_plus} ...wℕ
| |
| .\
| . 7. u : ℕk
| . ⊢  ∃m:ℕ. (Π u:ℕk. h(u)) = h(u)·m ∈ ℕ
| . | by Inst:
| . |    Thm*  ∀a,b:ℤ, e:({a..b−}→ℤ), i:{a..b−}.
| . |          e(i) | (Π i:{a..b−}. e(i))
| . |    {components_divide_iter_mul}
| . |    Using:[0 | k | e(u):= h(u) | u]
| . |
| . 8. h(u) | (Π i:ℕk. h(i))
| . ⊢  ∃m:ℕ. (Π u:ℕk. h(u)) = h(u)·m ∈ ℕ
| . | by Rewrite-1 by
| . |    Thm*  ∀a,b:ℕ. a | b ⟺ (∃c:ℕ. b = a·c)
| . |    {divides_def_on_nat} ...wℕ
| . |
```

```
| . 9. ∃c:ℕ. (Π i:ℕk. h(i)) = h(u)·c
| . ⊢  ∃m:ℕ. (Π u:ℕk. h(u)) = h(u)·m ∈ ℕ  by SimilarTo: Hyp:-1 ...ℕ
| . |
| . 9. c : ℕ
| . 10. (Π i:ℕk. h(i)) = h(u)·c
| . 11. x : ℕ+
| . 12. y : ℕ+
| . ⊢  x·y ∈ ℕ+
| .   by BackThru: Thm*  ∀x,y:ℕ+. x·y ∈ ℕ+    {mul_nat_plus} ...wℕ
|
.\
. 7. i : ℕ+
. 8. u : ℕk
. 9. ∃m:ℕ. i = h(u)·m ∈ ℕ
. ⊢  f{^0i}(u) = u  by Analyze-1
. |
. 9. m : ℕ
. 10. i = h(u)·m ∈ ℕ
. ⊢  f{^0i}(u) = u
. | by Rewrite← by
. |    Thm*  ∀x:A, f:(A→A), i,j,k:ℕ.
. |          k = i·j ⇒ f{^0i}{^0j}(x) = f{^0k}(x) ∈ A
. |    {compose_iter_prod}
. |    Using:[i:= h(u) |
. |           j:= m |
. |           f:= f |
. |           k:= i |
. |           x:= u |
. |           A:= ℕk] ...aℕ
. |
. ⊢  f{^0h(u)}{^0m}(u) = u
. | by BackThru:
. |    Thm*  ∀f:(A→A), u:A. f(u) = u ⇒ (∀i:ℕ. f{^0i}(u) = u)
. |    {compose_iter_point_id} ...wℕ
. |
. ⊢  f{^0h(u)}(u) = u  by BackThru: Hyp:6 ...wℕ
END
```