```
least_characterized  THM

⊢  ∀k:ℕ, p:{p:(ℕk→𝔹)| ∃i:ℕk. p(i) }.
|  (least i:ℕ. p(i)) ∈ {i:ℕk| p(i) & (∀j:ℕi. ¬p(j)) }
| by Induction on k
|
|\
| .....basecase .....
| ⊢  ∀p:{p:(ℕ0→𝔹)| ∃i:ℕ0. p(i) }.
|    (least i:ℕ. p(i)) ∈ {i:ℕ0| p(i) & (∀j:ℕi. ¬p(j)) }
|   by ℕ0 is empty
|
.\
. .....upcase .....
. 1. k : ℤ
. 2. 0<k
. 3. ∀p:{p:(ℕ(k-1)→𝔹)| ∃i:ℕ(k-1). p(i) }.
. |  (least i:ℕ. p(i)) ∈ {i:ℕ(k-1)| p(i) & (∀j:ℕi. ¬p(j)) }
. ⊢  ∀p:{p:(ℕk→𝔹)| ∃i:ℕk. p(i) }.
. |  (least i:ℕ. p(i)) ∈ {i:ℕk| p(i) & (∀j:ℕi. ¬p(j)) }
. | by Compute
. |      ⌈least i:ℕ. p(i)⌉ *⇒ if p(0)→ 0 else (least i:ℕ. p(i+1))+1 fi
. |    THEN
. |    UnivCD THEN Analyze-1 THEN Analyze-1
. |
. 4. p : ℕk→𝔹
. 5. i : ℕk
. 6. p(i)
. ⊢  if p(0)→ 0 else (least i:ℕ. p(i+1))+1 fi
. |  ∈ {i:ℕk| p(i) & (∀j:ℕi. ¬p(j)) }  by SplitOnConclITE
. |
. |\
. | .....truecase .....
. | 7. p(0)
. | ⊢  0 ∈ {i:ℕk| p(i) & (∀j:ℕi. ¬p(j)) }  by Analyze
. |
. .\
. . .....falsecase .....
. . 7. ¬p(0)
. . ⊢  (least i:ℕ. p(i+1))+1 ∈ {i:ℕk| p(i) & (∀j:ℕi. ¬p(j)) }
. . | by Use:[p(i):= p(i+1)] Inst: Hyp:3
. . |
. . |\
. . | .....wf .....
. . | ⊢  (λi.p(i+1)) ∈ {p:(ℕ(k-1)→𝔹)| ∃i:ℕ(k-1). p(i) }  by Analyze
. . | |
. . | ⊢  ∃i:ℕ(k-1). p(i+1)  by Witness: i-1
. . | |
. . | |\
. . | | ⊢  0≤i-1  by i ≠ 0 ∈ ℕk  Asserted
. . | | |
. . | | .....assertion .....
. . | | ⊢  i ≠ 0 ∈ ℕk  by Analyze
. . | | |
. . | | 8. i = 0 ∈ ℕk
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. . | | ⊢  False  by Rewrite6 by Hyp:-1
. . | |
. . | .\
. . | . ⊢  p(i-1+1)  by Rewrite by i-1+1 = i ∈ ℕk
. . |
. . .\
. . . 8. (least i:ℕ. p(i+1)) ∈ {i:ℕ(k-1)| p(i+1) & (∀j:ℕi. ¬p(j+1)) }
. . . ⊢  (least i:ℕ. p(i+1))+1 ∈ {i:ℕk| p(i) & (∀j:ℕi. ¬p(j)) }
. . . | by Analyze-1
. . . |
. . . 8. (least i:ℕ. p(i+1)) ∈ ℕ(k-1)
. . . 9. p((least i:ℕ. p(i+1))+1)
. . . 10. ∀j:ℕ(least i:ℕ. p(i+1)). ¬p(j+1)
. . . ⊢  (least i:ℕ. p(i+1))+1 ∈ {i:ℕk| p(i) & (∀j:ℕi. ¬p(j)) }
. . . | by (least i:ℕ. p(i+1)) ∈ ℤ  By BackThru: ℕ(k-1) ⊆ ℤ
. . . |    {← for type checking }
. . . |
. . . |    THEN
. . . |    Analyze
. . . |
. . . 11. (least i:ℕ. p(i+1)) ∈ ℤ
. . . 12. j : ℕ((least i:ℕ. p(i+1))+1)
. . . ⊢  ¬p(j)  by Decide: j = 0 ∈ ℕk
. . . |
. . . |\
. . . | 13. j = 0 ∈ ℕk
. . . | ⊢  ¬p(j)  by Rewrite by Hyp:-1
. . . |
. . . .\
. . . . 13. ¬j = 0 ∈ ℕk
. . . . ⊢  ¬p(j)  by Witness10: j-1 THEN Rewrite by j = j-1+1 ∈ ℕk
END
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