```
least_exists  THM

⊢  ∀k:ℕ, P:{P:(ℕk→Prop)| ∃i:ℕk. P(i) }.
|  (∀i:ℕk. Dec(P(i))) ⇒ (∃∈{i:ℕk| P(i) & (∀j:ℕi. ¬P(j)) })
| by UnivCD THEN Analyze2
|
1. k : ℕ
2. P : ℕk→Prop
3. ∃i:ℕk. P(i)  [not for witness]
4. ∀i:ℕk. Dec(P(i))
⊢  ∃∈{i:ℕk| P(i) & (∀j:ℕi. ¬P(j)) }
| by Rewrite'-1 by
|    Thm*  (∀x:T. Dec(E(x))) ⟺ (∃f:(T→𝔹). ∀x:T. f(x) ⟺ E(x))
|    {dec_pred_iff_some_boolfun}
|    THEN
|    New:p Analyze-1
|
4. p : ℕk→𝔹
5. ∀i:ℕk. p(i) ⟺ P(i)
⊢  ∃∈{i:ℕk| P(i) & (∀j:ℕi. ¬P(j)) }
| by Inst:
|    Thm*  ∀k:ℕ, p:{p:(ℕk→𝔹)| ∃i:ℕk. p(i) }.
|           (least i:ℕ. p(i)) ∈ {i:ℕk| p(i) & (∀j:ℕi. ¬p(j)) }
|    {least_characterized}
|    Using:[k | p]
|
|\
| .....wf .....
| 3. ∃i:ℕk. P(i)
| 4. p : ℕk→𝔹
| 5. ∀i:ℕk. p(i) ⟺ P(i)
| ⊢  p ∈ {p:(ℕk→𝔹)| ∃i:ℕk. p(i) }  by Analyze
| |
| .....set predicate .....
| ⊢  ∃i:ℕk. p(i)  by Rewrite by Hyp:5
|
.\
. 6. (least i:ℕ. p(i)) ∈ {i:ℕk| p(i) & (∀j:ℕi. ¬p(j)) }
. ⊢  ∃∈{i:ℕk| P(i) & (∀j:ℕi. ¬P(j)) }  by Witness: least i:ℕ. p(i)
. |
. 3. ∃i:ℕk. P(i)
. 4. p : ℕk→𝔹
. 5. ∀i:ℕk. p(i) ⟺ P(i)
. 6. (least i:ℕ. p(i)) ∈ {i:ℕk| p(i) & (∀j:ℕi. ¬p(j)) }
. ⊢  (least i:ℕ. p(i)) ∈ {i:ℕk| P(i) & (∀j:ℕi. ¬P(j)) }
. | by CoerceToType6: {i:ℕk| P(i) & (∀j:ℕi. ¬P(j)) }
. |
. .....fun wf .....
. 7. u : {i:ℕk| p(i) & (∀j:ℕi. ¬p(j)) }
. ⊢  u ∈ {i:ℕk| P(i) & (∀j:ℕi. ¬P(j)) }  by Analyze-1 THEN Analyze
. |
. .....set predicate .....
. 7. u : ℕk
. 8. p(u) & (∀j:ℕu. ¬p(j))
. ⊢  P(u) & (∀j:ℕu. ¬P(j))  by Rewrite by Hyp:5
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END
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