```
least_exists2  THM

⊢  ∀P:{P:(ℕ→Prop)| ∃i:ℕ. P(i) }.
|  (∀i:ℕ. Dec(P(i))) ⇒ (∃∈{i:ℕ| P(i) & (∀j:ℕ. j<i ⇒ ¬P(j)) })
| by UnivCD THEN Analyze1
|
1. P : ℕ→Prop
2. ∃i:ℕ. P(i)  [not for witness]
3. ∀i:ℕ. Dec(P(i))
⊢  ∃∈{i:ℕ| P(i) & (∀j:ℕ. j<i ⇒ ¬P(j)) }
| by Rewrite'-1 by
|    Thm*  (∀x:T. Dec(E(x))) ⟺ (∃f:(T→𝔹). ∀x:T. f(x) ⟺ E(x))
|    {dec_pred_iff_some_boolfun}
|    THEN
|    New:p Analyze-1
|
3. p : ℕ→𝔹
4. ∀i:ℕ. p(i) ⟺ P(i)
⊢  ∃∈{i:ℕ| P(i) & (∀j:ℕ. j<i ⇒ ¬P(j)) }
| by Inst:
|    Thm*  ∀p:{p:(ℕ→𝔹)| ∃i:ℕ. p(i) }.
|          (least i:ℕ. p(i)) ∈ {i:ℕ| p(i) & (∀j:ℕ. j<i ⇒ ¬p(j)) }
|    {least_characterized2}
|    Using:[p]
|
|\
| .....wf .....
| 2. ∃i:ℕ. P(i)
| 3. p : ℕ→𝔹
| 4. ∀i:ℕ. p(i) ⟺ P(i)
| ⊢  p ∈ {p:(ℕ→𝔹)| ∃i:ℕ. p(i) }  by Analyze
| |
| .....set predicate .....
| ⊢  ∃i:ℕ. p(i)  by Rewrite by Hyp:4
|
.\
. 5. (least i:ℕ. p(i)) ∈ {i:ℕ| p(i) & (∀j:ℕ. j<i ⇒ ¬p(j)) }
. ⊢  ∃∈{i:ℕ| P(i) & (∀j:ℕ. j<i ⇒ ¬P(j)) }
. | by Witness: least i:ℕ. p(i)
. |
. 2. ∃i:ℕ. P(i)
. 3. p : ℕ→𝔹
. 4. ∀i:ℕ. p(i) ⟺ P(i)
. 5. (least i:ℕ. p(i)) ∈ {i:ℕ| p(i) & (∀j:ℕ. j<i ⇒ ¬p(j)) }
. ⊢  (least i:ℕ. p(i)) ∈ {i:ℕ| P(i) & (∀j:ℕ. j<i ⇒ ¬P(j)) }
. | by CoerceToType5: {i:ℕ| P(i) & (∀j:ℕ. j<i ⇒ ¬P(j)) }
. |
. .....fun wf .....
. 6. u : {i:ℕ| p(i) & (∀j:ℕ. j<i ⇒ ¬p(j)) }
. ⊢  u ∈ {i:ℕ| P(i) & (∀j:ℕ. j<i ⇒ ¬P(j)) }
. | by Analyze-1 THEN Analyze
. |
. .....set predicate .....
. 6. u : ℕ
. 7. p(u) & (∀j:ℕ. j<u ⇒ ¬p(j))
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. ⊢  P(u) & (∀j:ℕ. j<u ⇒ ¬P(j))  by Rewrite by Hyp:4
END
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