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nat_to_nat_pair_surj  THM

⊢  Surj(ℕ; ℕ×ℕ; nat_to_nat_pair)  by Def
|
⊢  ∀b:(ℕ×ℕ). ∃a:ℕ. nat_to_nat_pair(a) = b
| by ∀k,x,y:ℕ. x+y = k ⇒ (∃a:ℕ. nat_to_nat_pair(a) = <x,y>)
|    Asserted
|
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| .....assertion .....
| ⊢  ∀k,x,y:ℕ. x+y = k ⇒ (∃a:ℕ. nat_to_nat_pair(a) = <x,y>)
| | by 2 Times (CompNatInd Concl)
| |
| 1. k : ℕ
| 2. ∀k1:ℕ.
| |  k1<k ⇒ (∀x,y:ℕ. x+y = k1 ⇒ (∃a:ℕ. nat_to_nat_pair(a) = <x,y>))
| 3. x : ℕ
| 4. ∀x1:ℕ.
| |  x1<x ⇒ (∀y:ℕ. x1+y = k ⇒ (∃a:ℕ. nat_to_nat_pair(a) = <x1,y>))
| 5. y : ℕ
| 6. x+y = k
| ⊢  ∃a:ℕ. nat_to_nat_pair(a) = <x,y>
| | by Decide: x = 0 THENL [Decide: y = 0;Id]
| |
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| | 7. x = 0
| | 8. y = 0
| | ⊢  ∃a:ℕ. nat_to_nat_pair(a) = <x,y>  by Witness: 0
| | |
| | ⊢  nat_to_nat_pair(0) = <x,y>
| |   by Compute ⌈nat_to_nat_pair(0)⌉ *⇒ <0,0> THEN Analyze
| |
| |\
| | 7. x = 0
| | 8. ¬y = 0
| | ⊢  ∃a:ℕ. nat_to_nat_pair(a) = <x,y>
| | | by Use:[y-1 | y-1 | 0] Inst: {outer inductive hyp } Hyp:2
| | |    THEN
| | |    New:n Analyze-1
| | |
| | 9. n : ℕ
| | 10. nat_to_nat_pair(n) = <y-1,0> ∈ ℕ×ℕ
| | ⊢  ∃a:ℕ. nat_to_nat_pair(a) = <x,y>  by Witness: n+1
| | |
| | ⊢  nat_to_nat_pair(n+1) = <x,y>
| | | by Compute ⌈nat_to_nat_pair(n+1)⌉ THEN SplitITE Concl
| | |    THEN
| | |    Subst: n+1-1 = n ∈ ℕ
| | |
| | 11. n+1 ≠ 0
| | ⊢  next_nat_pair(nat_to_nat_pair(n)) = <x,y>  by Subst: Hyp:10
| | |
| | ⊢  next_nat_pair(<y-1,0>) = <x,y>
| |   by Compute ⌈next_nat_pair(<y-1,0>)⌉ *⇒ <0,y-1+1> THEN Analyze
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| . 7. ¬x = 0
| . ⊢  ∃a:ℕ. nat_to_nat_pair(a) = <x,y>
| . | by Use:[x-1 | y+1] Inst: {inner inductive hyp } Hyp:4
| . |    THEN
| . |    New:n Analyze-1
| . |
| . 8. n : ℕ
| . 9. nat_to_nat_pair(n) = <x-1,y+1> ∈ ℕ×ℕ
| . ⊢  ∃a:ℕ. nat_to_nat_pair(a) = <x,y>  by Witness: n+1
| . |
| . ⊢  nat_to_nat_pair(n+1) = <x,y>
| . | by Compute ⌈nat_to_nat_pair(n+1)⌉ THEN SplitITE Concl
| . |
| . .....falsecase .....
| . ⊢  next_nat_pair(nat_to_nat_pair(n+1-1)) = <x,y>
| . | by Subst: n+1-1 = n ∈ ℕ
| . |
| . ⊢  next_nat_pair(nat_to_nat_pair(n)) = <x,y>  by Subst: Hyp:9
| . |
| . ⊢  next_nat_pair(<x-1,y+1>) = <x,y>
| .   by Compute
| .        ⌈next_nat_pair(<x-1,y+1>)⌉
| .        *⇒
| .        if y+1=₂0→ <0,x-1+1> else <x-1+1,y+1-1> fi
| .      THEN
| .      SplitITE Concl
|
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. 1. ∀k,x,y:ℕ. x+y = k ⇒ (∃a:ℕ. nat_to_nat_pair(a) = <x,y>)
. ⊢  ∀b:(ℕ×ℕ). ∃a:ℕ. nat_to_nat_pair(a) = b
. | by Analyze THEN New:[x | y] Analyze-1
. |
. 2. x : ℕ
. 3. y : ℕ
. ⊢  ∃a:ℕ. nat_to_nat_pair(a) = <x,y>  by Use:[x+y] BackThru: Hyp:1
END
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