```
next_nat_pair_inj  THM

⊢  Inj(ℕ×ℕ; ℕ×ℕ; next_nat_pair)
| by Analyze
|    THEN
|    New:[x1 | y1] Analyze-3 THEN New:[x2 | y2] Analyze-2
|
1. x1 : ℕ
2. y1 : ℕ
3. x2 : ℕ
4. y2 : ℕ
5. next_nat_pair(<x1,y1>) = next_nat_pair(<x2,y2>)
⊢  <x1,y1> = <x2,y2>
| by Compute-1
|      (⌈next_nat_pair(<x1,y1>)⌉
|       *⇒
|       if y1=₂0→ <0,x1+1> else <x1+1,y1-1> fi)
|      =
|      (⌈next_nat_pair(<x2,y2>)⌉
|       *⇒
|       if y2=₂0→ <0,x2+1> else <x2+1,y2-1> fi)
|      ∈ ℕ×ℕ
|    THEN
|    2 Times (SplitITE Hyp:5)
|    THEN
|    Analyze5 THEN OnAllHyps Reduce THEN RemoveImpossibleCasesBy Auto
|
|\
| 5. 0 = 0 ∈ ℕ
| 6. x1+1 = x2+1 ∈ ℕ
| 7. y1 = 0
| 8. y2 = 0
| ⊢  <x1,y1> = <x2,y2>  by Analyze
|
.\
. 5. x1+1 = x2+1 ∈ ℕ
. 6. y1-1 = y2-1 ∈ ℕ
. 7. y1 ≠ 0
. 8. y2 ≠ 0
. ⊢  <x1,y1> = <x2,y2>  by Analyze
END
```