```
no_finite_model  THM

⊢  ¬(∃A:Type, R:(A→A→Prop).
|    (∃∈A) & (Trans x,y:A. R(x;y)) & (∀x:A. ∃y:A. R(x;y))
|    & ¬(∃x:A. R(x;x))
|    & Finite(A))  by SideProof 1
|
1. A : Type
2. R : A→A→Prop
3. ∃∈A
4. Trans x,y:A. R(x;y)
5. ∀x:A. ∃y:A. R(x;y)
6. Finite(A)
⊢  ∃x:A. R(x;x)
| by Inst:
|    Thm*  ∀R:(A→A→Prop).
|          (∃∈A) & (Trans x,y:A. R(x;y)) & (∀x:A. ∃y:A. R(x;y))
|          ⇒
|          (∀k:ℕ⁺. ∃f:(ℕk→A). ∀i,j:ℕk. i<j ⇒ R(f(i);f(j)))
|    {no_finite_model_lemma}
|    Using:[A | R]
|
7. ∀k:ℕ⁺. ∃f:(ℕk→A). ∀i,j:ℕk. i<j ⇒ R(f(i);f(j))
⊢  ∃x:A. R(x;x)  by Analyze6
|
6. n : ℕ
7. A ∼ ℕn
8. ∀k:ℕ⁺. ∃f:(ℕk→A). ∀i,j:ℕk. i<j ⇒ R(f(i);f(j))
⊢  ∃x:A. R(x;x)  by Witness8: n+1 THEN Analyze-1
|
9. f : ℕ(n+1)→A
10. ∀i,j:ℕ(n+1). i<j ⇒ R(f(i);f(j))
⊢  ∃x:A. R(x;x)  by New:e Analyze7 THEN Analyze8
|
7. e : A→ℕn
8. g : ℕn→A
9. InvFuns(A;ℕn;e;g)
10. ∀k:ℕ⁺. ∃f:(ℕk→A). ∀i,j:ℕk. i<j ⇒ R(f(i);f(j))
11. f : ℕ(n+1)→A
12. ∀i,j:ℕ(n+1). i<j ⇒ R(f(i);f(j))
⊢  ∃x:A. R(x;x)
| by FwdThru:
|    Thm*  ∀m,k:ℕ, f:(ℕm→ℕk). k<m ⇒ (∃x,y:ℕm. x ≠ y & f(x) = f(y))
|    {pigeon_hole}
|    on [ n<n+1 ]
|    Using:[e o f]
|
13. ∃x,y:ℕ(n+1). x ≠ y & (e o f)(x) = (e o f)(y)
⊢  ∃x:A. R(x;x)
| by ∃x,y:ℕ(n+1). x<y & (e o f)(x) = (e o f)(y) ∈ ℤ  By SideProof 2
|    THEN
|    ExistHD Hyp:-1
|
14. x : ℕ(n+1)
15. y : ℕ(n+1)
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16. x<y
17. (e o f)(x) = (e o f)(y)
⊢  ∃x:A. R(x;x)  by Analyze9
|
18. ∀x:A. g(e(x)) = x ∈ A
⊢  ∃x:A. R(x;x)
| by ApFun: g to: (e o f)(x) = (e o f)(y) ∈ ℤ THEN Reduce Hyp:-1
|
19. g(e(f(x))) = g(e(f(y)))
⊢  ∃x:A. R(x;x)  by Rewrite-1 by Hyp:18
|
20. f(x) = f(y)
⊢  ∃x:A. R(x;x)  by FwdThru: Hyp:12 on [ x<y ]
|
21. R(f(x);f(y))
⊢  ∃x:A. R(x;x)  by Subst-1: f(x) = f(y) ∈ A
|
21. R(f(y);f(y))
⊢  ∃x:A. R(x;x)  by Witness: f(y)
END
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no_finite_model Side Proof 1  THM

⊢  ¬(∃A:Type, R:(A→A→Prop).
|    (∃∈A) & (Trans x,y:A. R(x;y)) & (∀x:A. ∃y:A. R(x;y))
|    & ¬(∃x:A. R(x;x))
|    & Finite(A))  by Analyze
|
1. ∃A:Type, R:(A→A→Prop).
|  (∃∈A) & (Trans x,y:A. R(x;y)) & (∀x:A. ∃y:A. R(x;y))
|  & ¬(∃x:A. R(x;x))
|  & Finite(A)
⊢  False  by ExistHD Hyp:-1
|
1. A : Type
2. R : A→A→Prop
3. ∃∈A
4. Trans x,y:A. R(x;y)
5. ∀x:A. ∃y:A. R(x;y)
6. ¬(∃x:A. R(x;x))
7. Finite(A)
⊢  False  by Analyze-2
|
6. Finite(A)
⊢  ∃x:A. R(x;x)  by <Unproved PREMISE>
END
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no_finite_model Side Proof 2  THM

.....assertion .....
1. A : Type
2. R : A→A→Prop
3. ∃∈A
4. Trans x,y:A. R(x;y)
5. ∀x:A. ∃y:A. R(x;y)
6. k : ℕ
7. e : A→ℕk
8. g : ℕk→A
9. InvFuns(A;ℕk;e;g)
10. ∀k:ℕ⁺. ∃f:(ℕk→A). ∀i,j:ℕk. i<j ⇒ R(f(i);f(j))
11. f : ℕ(k+1)→A
12. ∀i,j:ℕ(k+1). i<j ⇒ R(f(i);f(j))
13. ∃x,y:ℕ(k+1). x ≠ y & (e o f)(x) = (e o f)(y)
⊢  ∃x,y:ℕ(k+1). x<y & (e o f)(x) = (e o f)(y)  by ExistHD Hyp:13
|
13. x : ℕ(k+1)
14. y : ℕ(k+1)
15. x ≠ y
16. (e o f)(x) = (e o f)(y)
⊢  ∃x,y:ℕ(k+1). x<y & (e o f)(x) = (e o f)(y)  by Decide: x<y
|
|\
| 17. x<y
| ⊢  ∃x,y:ℕ(k+1). x<y & (e o f)(x) = (e o f)(y)  by Witness: x | y
|
.\
. 17. ¬x<y
. ⊢  ∃x,y:ℕ(k+1). x<y & (e o f)(x) = (e o f)(y)  by Witness: y | x
END
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