```
no_finite_model_lemma  THM

⊢  ∀A:Type, R:(A→A→Prop).
|  (∃∈A) & (Trans x,y:A. R(x;y)) & (∀x:A. ∃y:A. R(x;y))
|  ⇒
|  (∀k:ℕ⁺. ∃f:(ℕk→A). ∀i,j:ℕk. i<j ⇒ R(f(i);f(j)))  by Auto
|
1. A : Type
2. R : A→A→Prop
3. ∃∈A
4. Trans x,y:A. R(x;y)
5. ∀x:A. ∃y:A. R(x;y)
6. k : ℕ⁺
⊢  ∃f:(ℕk→A). ∀i,j:ℕk. i<j ⇒ R(f(i);f(j))  by NSubsetInd Hyp:-1
|
|\
| .....basecase .....
| 6. k : ℤ
| 7. 0<k
| 8. 1 = k
| ⊢  ∃f:(ℕk→A). ∀i,j:ℕk. i<j ⇒ R(f(i);f(j))
|   by Rename3: a THEN Witness: λz.a
|
.\
. .....upcase .....
. 6. k : ℤ
. 7. 1<k
. 8. ∃f:(ℕ(k-1)→A). ∀i,j:ℕ(k-1). i<j ⇒ R(f(i);f(j))
. ⊢  ∃f:(ℕk→A). ∀i,j:ℕk. i<j ⇒ R(f(i);f(j))  by Analyze-1
. |
. 8. f : ℕ(k-1)→A
. 9. ∀i,j:ℕ(k-1). i<j ⇒ R(f(i);f(j))
. ⊢  ∃f:(ℕk→A). ∀i,j:ℕk. i<j ⇒ R(f(i);f(j))
. | by Witness5: f(k-2) {← the last member of sequence f }
. |    THEN
. |    Analyze-1
. |
. 10. y : A
. 11. R(f(k-2);y)
. ⊢  ∃f:(ℕk→A). ∀i,j:ℕk. i<j ⇒ R(f(i);f(j))
. | by Witness: λz.if z<₂k-1→ f(z) else y fi
. |
. |\
. | .....wf .....
. | ⊢  (λz.if z<₂k-1→ f(z) else y fi) ∈ ℕk→A
. |   by Analyze THEN SplitITE Concl
. |
. .\
. . ⊢  ∀i,j:ℕk.
. . |  i<j
. . |  ⇒
. . |  R((λz.if z<₂k-1→ f(z) else y fi)(i)
. . |   ;(λz.if z<₂k-1→ f(z) else y fi)(j))  by Reduce Concl
. . |
. . 12. i : ℕk
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. . 13. j : ℕk
. . 14. i<j
. . ⊢  R(if i<₂k-1→ f(i) else y fi;if j<₂k-1→ f(j) else y fi)
. . | by 2 Times SplitOnConclITE
. . |
. . |\
. . | .....truecase .....
. . | 15. i<k-1
. . | 16. j<k-1
. . | ⊢  R(f(i);f(j))  by BackThru: Hyp:9
. . |
. . .\
. . . .....falsecase .....
. . . 15. i<k-1
. . . 16. k-1≤j
. . . ⊢  R(f(i);y)
. . . | by BackThru: Trans x,y:A. R(x;y) Using:[f(k-2)] ;
. . . |    Decide: i = k-2
. . . |
. . . |\
. . . | 17. i = k-2
. . . | ⊢  R(f(i);y)
. . . |   by ApFun: f to: i = k-2 THEN Subst: f(i) = f(k-2) ∈ A
. . . |
. . . .\
. . . . 17. ¬i = k-2
. . . . ⊢  R(f(i);y)  by BackThru: Trans x,y:A. R(x;y) Using:[f(k-2)]
. . . . |
. . . . ⊢  R(f(i);f(k-2))  by BackThru: Hyp:9
END
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