```
nsub_bij_least_preimage_inverse  THM

⊢  ∀f:(ℕa bij→ ℕa). InvFuns(ℕa;ℕa;f;λy.least x:ℕ. f(x)=2y)  by Auto
|
1. a : ℕ
2. f : ℕa bij→ ℕa
⊢  InvFuns(ℕa;ℕa;f;λy.least x:ℕ. f(x)=2y)
| by f ∈ ℕa onto→ ℕa
|      By BackThru:
|         Thm*  (A bij→ B) ⊆ (A onto→ B)    {bijtype_sub_surjtype}
|    THEN
|    f ∈ ℕa bij→ ℕa  Asserted THEN Analyze-1
|
3. f ∈ ℕa onto→ ℕa
4. f ∈ ℕa→ℕa
5. Bij(ℕa; ℕa; f)  [not for witness]
⊢  InvFuns(ℕa;ℕa;f;λy.least x:ℕ. f(x)=2y)
| by Use:[a | a | f]
|    Inst:
|    Thm*  ∀f:(ℕa onto→ ℕb). (λy.least x:ℕ. f(x)=2y) ∈ ℕb inj→ ℕa
|    {nsub_surj_imp_a_rev_inj}
|    THEN
|    Analyze-1
|
6. (λy.least x:ℕ. f(x)=2y) ∈ ℕa→ℕa
7. Inj(ℕa; ℕa; λy.least x:ℕ. f(x)=2y)  [not for witness]
⊢  InvFuns(ℕa;ℕa;f;λy.least x:ℕ. f(x)=2y)
| by Analyze THEN Reduce Concl
|
|\
| 5. Bij(ℕa; ℕa; f)
| 6. (λy.least x:ℕ. f(x)=2y) ∈ ℕa→ℕa
| 7. Inj(ℕa; ℕa; λy.least x:ℕ. f(x)=2y)
| 8. x : ℕa
| ⊢  (least x@0:ℕ. f(x@0)=2f(x)) = x
| | by Use:[a | a | f | f(x)]
| |    Inst:
| |    Thm*  ∀f:(ℕa onto→ ℕb), y:ℕb. f(least x:ℕ. f(x)=2y) = y
| |    {nsub_surj_least_preimage_works}
| |
| 9. f(least x@0:ℕ. f(x@0)=2f(x)) = f(x)
| ⊢  (least x@0:ℕ. f(x@0)=2f(x)) = x
| | by Guarding (least [var]:ℕ. <bool>) BackThru: Inj(ℕa; ℕa; f)
| |
| |\
| | .....assertion .....
| | ⊢  Inj(ℕa; ℕa; f)  by BackThru: Bij(ℕa; ℕa; f)
| |
| .\
| . ⊢  (least x@0:ℕ. f(x@0)=2f(x)) ∈ ℕa
| .   by BackThru:
| .      Thm*  ∀f:(ℕa onto→ ℕb), y:ℕb. (least x:ℕ. f(x)=2y) ∈ ℕa
| .      {nsub_surj_least_preimage_total}
|
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. 5. Bij(ℕa; ℕa; f)
. 6. (λy.least x:ℕ. f(x)=₂y) ∈ ℕa→ℕa
. 7. Inj(ℕa; ℕa; λy.least x:ℕ. f(x)=₂y)
. 8. y : ℕa
. ⊢  f(least x:ℕ. f(x)=₂y) = y
.    by Use:[a]
.       BackThru:
.       Thm*  ∀f:(ℕa onto→ ℕb), y:ℕb. f(least x:ℕ. f(x)=₂y) = y
.       {nsub_surj_least_preimage_works}
END
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