```
nsub_bool  THM

⊢  𝔹 ∼ ℕ2
| by Witness: λb.if b→ 1 else 0 fi | λi.if i=0→ false_2 ; true_2 fi
|
⊢  InvFuns(𝔹;ℕ2;λb.if b→ 1 else 0 fi;λi.if i=0→ false_2 ; true_2 fi)
| by Analyze THEN Reduce Concl
|
|\
| 1. x : 𝔹
| ⊢  if if x→ 1 else 0 fi=0→ false_2 ; true_2 fi = x
|   by CaseITE Concl THEN Reduce Concl
|
.\
. 1. y : ℕ2
. ⊢  if if y=0→ false_2 ; true_2 fi→ 1 else 0 fi = y ∈ ℕ2
. | by Decide: y = 0 ∈ ℕ2 | y = 1 ∈ ℕ2
. |
. |\
. | 2. y = 0 ∈ ℕ2
. | 3. ¬y = 1 ∈ ℕ2
. | ⊢  if if y=0→ false_2 ; true_2 fi→ 1 else 0 fi = y ∈ ℕ2
. |   by Subst: Hyp:2 THEN Reduce Concl
. |
. .\
. . 2. ¬y = 0 ∈ ℕ2
. . 3. y = 1 ∈ ℕ2
. . ⊢  if if y=0→ false_2 ; true_2 fi→ 1 else 0 fi = y ∈ ℕ2
. .   by Subst: Hyp:3 THEN Reduce Concl
END
```