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nsub_discr_range_surj  THM

⊢  ∀A:Type.
|  (A Discrete)
|  ⇒
|  (∀k:ℕ, f:(ℕk→A).
|   {a:A| ∃i:ℕk. a = f(i) } ∈ Type
|   & f ∈ ℕk→{a:A| ∃i:ℕk. a = f(i) }
|   & Surj(ℕk; {a:A| ∃i:ℕk. a = f(i) }; f))
| by Guarding (<prop> & <prop>) Auto
|
1. A : Type
2. A Discrete
3. k : ℕ
4. f : ℕk→A
⊢  {a:A| ∃i:ℕk. a = f(i) } ∈ Type
|  & f ∈ ℕk→{a:A| ∃i:ℕk. a = f(i) }
|  & Surj(ℕk; {a:A| ∃i:ℕk. a = f(i) }; f)  by 2 Times Analyze
|
|\
| 5. {a:A| ∃i:ℕk. a = f(i) } ∈ Type
| ⊢  f ∈ ℕk→{a:A| ∃i:ℕk. a = f(i) }
| | by FunExtensionality THEN Analyze
| |
| .....set predicate .....
| 6. x : ℕk
| ⊢  ∃i:ℕk. f(x) = f(i)  by Witness: x
|
.\
. 5. {a:A| ∃i:ℕk. a = f(i) } ∈ Type
. 6. f ∈ ℕk→{a:A| ∃i:ℕk. a = f(i) }
. ⊢  Surj(ℕk; {a:A| ∃i:ℕk. a = f(i) }; f)  by Analyze
. |
. 7. b : {a:A| ∃i:ℕk. a = f(i) }
. ⊢  ∃a:ℕk. f(a) = b  by Analyze-1
. |
. 7. b : A
. 8. ∃i:ℕk. b = f(i)  [not for witness]
. ⊢  ∃a:ℕk. f(a) = b ∈ {a:A| ∃i:ℕk. a = f(i) ∈ A }
. | by FwdThru:
. |    Thm*  (A Discrete) ⟺ (∃e:(A→A→𝔹). ∀x,y:A. (x e y) ⟺ x = y)
. |    {discrete_vs_bool}
. |    on [ A Discrete ]
. |    THEN
. |    Analyze-1
. |
. 9. e : A→A→𝔹
. 10. ∀x,y:A. (x e y) ⟺ x = y
. ⊢  ∃a:ℕk. f(a) = b ∈ {a:A| ∃i:ℕk. a = f(i) ∈ A }
. | by (λj.(f(j)) e b) ∈ {p:(ℕk→𝔹)| ∃i:ℕk. p(i) }  Asserted
. |    THEN
. |    Inst:
. |    Thm*  ∀k:ℕ, p:{p:(ℕk→𝔹)| ∃i:ℕk. p(i) }. (least i:ℕ. p(i)) ∈ ℕk
. |    {least_wf}
. |    Using:[k | λj.(f(j)) e b]
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. |    THEN
. |    (ReduceSOAps Hyp:-1)
. |
. |\
. | .....assertion .....
. | ⊢  (λj.(f(j)) e b) ∈ {p:(ℕk→𝔹)| ∃i:ℕk. p(i) }
. | | by Analyze THEN ReduceSOAps Concl
. | |
. | 8. ∃i:ℕk. b = f(i)
. | 9. e : A→A→𝔹
. | 10. ∀x,y:A. (x e y) ⟺ x = y
. | ⊢  ∃i:ℕk. (f(i)) e b  by Rewrite by Hyp:-1
. | |
. | ⊢  ∃i:ℕk. f(i) = b  by SimilarTo: Hyp:8
. |
. .\
. . 11. (λj.(f(j)) e b) ∈ {p:(ℕk→𝔹)| ∃i:ℕk. p(i) }
. . 12. (least i:ℕ. (f(i)) e b) ∈ ℕk
. . ⊢  ∃a:ℕk. f(a) = b  by Witness: least i:ℕ. (f(i)) e b
. . |
. . 8. ∃i:ℕk. b = f(i)
. . 9. e : A→A→𝔹
. . 10. ∀x,y:A. (x e y) ⟺ x = y
. . 11. (λj.(f(j)) e b) ∈ {p:(ℕk→𝔹)| ∃i:ℕk. p(i) }
. . 12. (least i:ℕ. (f(i)) e b) ∈ ℕk
. . ⊢  f(least i:ℕ. (f(i)) e b) = b
. . | by Inst:
. . |    Thm*  ∀k:ℕ, p:{p:(ℕk→𝔹)| ∃i:ℕk. p(i) }. p(least i:ℕ. p(i))
. . |    {least_satisfies}
. . |    Using:[k | λj.(f(j)) e b]
. . |    THEN
. . |    ReduceSOAps Hyp:-1
. . |
. . 13. (f(least i:ℕ. (f(i)) e b)) e b
. . ⊢  f(least i:ℕ. (f(i)) e b) = b  by Rewrite-1 by Hyp:10
. . |
. . 14. f(least i:ℕ. (f(i)) e b) = b
. . ⊢  f(least i:ℕ. (f(i)) e b) = b  by Analyze
. . |
. . .....set predicate .....
. . ⊢  ∃i:ℕk. f(least i:ℕ. (f(i)) e b) = f(i)
. .   by SimilarTo: ∃i:ℕk. b = f(i)
END
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