nsub_inj_fill_typing THM

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⊢  ∀a:ℕ+, b:ℕ, j:ℕb, f:(ℕ(a-1) inj→ {x:ℕb| ¬x = j }).
|  (λi.if i=₂a-1→ j else f(i) fi) ∈ ℕa inj→ ℕb  by Auto
|
1. a : ℕ+
2. b : ℕ
3. j : ℕb
4. f : ℕ(a-1) inj→ {x:ℕb| ¬x = j }
⊢  (λi.if i=₂a-1→ j else f(i) fi) ∈ ℕa inj→ ℕb  by Analyze-1
|
4. f : ℕ(a-1)→{x:ℕb| ¬x = j }
5. Inj(ℕ(a-1); {x:ℕb| ¬x = j }; f)
⊢  (λi.if i=₂a-1→ j else f(i) fi) ∈ ℕa inj→ ℕb
| by (λi.if i=₂a-1→ j else f(i) fi) ∈ ℕa→ℕb  Asserted THEN Analyze
|
|\
| .....assertion .....
| ⊢  (λi.if i=₂a-1→ j else f(i) fi) ∈ ℕa→ℕb  by Analyze
| |
| 6. i : ℕa
| ⊢  if i=₂a-1→ j else f(i) fi ∈ ℕb  by SplitITE Concl
|
.\
. .....set predicate .....
. 6. (λi.if i=₂a-1→ j else f(i) fi) ∈ ℕa→ℕb
. ⊢  Inj(ℕa; ℕb; λi.if i=₂a-1→ j else f(i) fi)  by Def THEN Reduce -1
. |
. 7. a1 : ℕa
. 8. a2 : ℕa
. 9. if a1=₂a-1→ j else f(a1) fi = if a2=₂a-1→ j else f(a2) fi
. ⊢  a1 = a2  by 2 Times (SplitITE 9)
. |
. |\
. | .....falsecase .....
. | 9. j = f(a2)
. | 10. a1 = a-1
. | 11. a2 ≠ a-1
. | ⊢  a1 = a2  by ApFun: f to: a2 THEN Analyze-1
. | |
. | 12. f(a2) ∈ ℕb
. | 13. ¬f(a2) = j
. | ⊢  a1 = a2  by Auto
. |
. |\
. | .....truecase .....
. | 9. f(a1) = j
. | 10. a1 ≠ a-1
. | 11. a2 = a-1
. | ⊢  a1 = a2  by {Similarly } ApFun: f to: a1 THEN Analyze-1
. |
. .\
. . .....falsecase .....
. . 9. f(a1) = f(a2)
. . 10. a1 ≠ a-1
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. . 11. a2 ≠ a-1
. . ⊢  a1 = a2
. . | by FwdThru: Hyp:5 on [ f(a1) = f(a2) ∈ {x:ℕb| ¬x = j } ]
. . |
. . .....assertion .....
. . ⊢  f(a1) = f(a2) ∈ {x:ℕb| ¬x = j ∈ ℤ }
. . | by Guarding (<fun>(<arg>)) Analyze
. . |
. . .....set predicate .....
. . ⊢  ¬f(a1) = j  by ApFun: f to: a1 THEN Analyze-1
END
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