```
nsub_inj_lop_typing  THM

⊢  ∀a:ℕ+ , b:ℕ, f:(ℕa inj→ ℕb). f ∈ ℕ(a-1) inj→ {x:ℕb| ¬x = f(a-1) }
| by Auto
|
1. a : ℕ+
2. b : ℕ
3. f : ℕa inj→ ℕb
⊢  f ∈ ℕ(a-1) inj→ {x:ℕb| ¬x = f(a-1) }  by Analyze-1
|
3. f : ℕa→ℕb
4. Inj(ℕa; ℕb; f)
⊢  f ∈ ℕ(a-1) inj→ {x:ℕb| ¬x = f(a-1) }
| by Analyze THEN FunExtensionality
|
|\
| 5. x : ℕ(a-1)
| ⊢  f(x) ∈ {x:ℕb| ¬x = f(a-1) }  by Analyze
| |
| .....set predicate .....
| ⊢  ¬f(x) = f(a-1)  by Witness4: x | a-1
| |
| 6. f(x) = f(a-1) ⇒ x = a-1 ∈ ℕa
| ⊢  ¬f(x) = f(a-1)  by SimilarTo: Hyp:-1
|
.\
. .....set predicate .....
. ⊢  Inj(ℕ(a-1); {x:ℕb| ¬x = f(a-1) }; f)  by Def THEN UnivCD
. |
. |\
. | 5. a1 : ℕ(a-1)
. | 6. a2 : ℕ(a-1)
. | 7. f(a1) = f(a2) ∈ {x:ℕb| ¬x = f(a-1) }
. | ⊢  a1 = a2  by FwdThru: Hyp:4 on [ Hyp:-1 ]
. |
. |\
. | 5. a1 : ℕ(a-1)
. | 6. ℕ(a-1)
. | ⊢  f(a1) ∈ {x:ℕb| ¬x = f(a-1) }  by Analyze THEN Analyze
. | |
. | 7. f(a1) = f(a-1)
. | ⊢  False  by FwdThru: Hyp:4 on [ f(a1) = f(a-1) ∈ ℕb ]
. |
. .\
. . 5. ℕ(a-1)
. . 6. a2 : ℕ(a-1)
. . ⊢  f(a2) ∈ {x:ℕb| ¬x = f(a-1) }
. .   by {Similarly }
. .      Analyze THEN Analyze
. .      THEN
. .      FwdThru: Hyp:4 on [ f(a2) = f(a-1) ∈ ℕb ]
END
```