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nsub_surj_imp_a_rev_inj_gen  THM

⊢  ∀B:Type, e:(B→B→𝔹).
|  IsEqFun(B;e)
|  ⇒
|  (∀a:ℕ, f:(ℕa onto→ B). (λy.least x:ℕ. (f(x)) e y) ∈ B inj→ ℕa)
| by Auto
|
1. B : Type
2. e : B→B→𝔹
3. IsEqFun(B;e)
4. a : ℕ
5. f : ℕa onto→ B
⊢  (λy.least x:ℕ. (f(x)) e y) ∈ B inj→ ℕa
| by f ∈ ℕa onto→ B  Asserted THEN Analyze5
|
5. f : ℕa→B
6. Surj(ℕa; B; f)
7. f ∈ ℕa onto→ B
⊢  (λy.least x:ℕ. (f(x)) e y) ∈ B inj→ ℕa
| by (λy,x. (f(x)) e y) ∈ B→{p:(ℕa→𝔹)| ∃i:ℕa. p(i) } |
|    (λy.least x:ℕ. (f(x)) e y) ∈ B→ℕa
|    Asserted
|
|\
| .....assertion .....
| ⊢  (λy,x. (f(x)) e y) ∈ B→{p:(ℕa→𝔹)| ∃i:ℕa. p(i) }
| | by Auto THEN Analyze THEN ReduceSOAps Concl
| |
| 8. y : B
| ⊢  ∃i:ℕa. (f(i)) e y  by Witness6: y
| |
| 9. ∃a:ℕa. f(a) = y
| ⊢  ∃i:ℕa. (f(i)) e y  by SimilarTo: Hyp:-1
| |
| 9. a1 : ℕa
| 10. f(a1) = y
| ⊢  (f(a1)) e y  by BackThru: 3
|
|\
| .....assertion .....
| 8. (λy,x. (f(x)) e y) ∈ B→{p:(ℕa→𝔹)| ∃i:ℕa. p(i) }
| ⊢  (λy.least x:ℕ. (f(x)) e y) ∈ B→ℕa
|   by Analyze
|      THEN
|      BackThru:
|      Thm*  ∀e:(B→B→𝔹).
|            IsEqFun(B;e)
|            ⇒
|            (∀a:ℕ, f:(ℕa onto→ B), y:B. (least x:ℕ. (f(x)) e y) ∈ ℕa)
|      {nsub_surj_least_preimage_total_gen}
|
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. 8. (λy,x. (f(x)) e y) ∈ B→{p:(ℕa→𝔹)| ∃i:ℕa. p(i) }
. 9. (λy.least x:ℕ. (f(x)) e y) ∈ B→ℕa
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. ⊢  (λy.least x:ℕ. (f(x)) e y) ∈ B inj→ ℕa  by Analyze
. |
. .....set predicate .....
. ⊢  Inj(B; ℕa; λy.least x:ℕ. (f(x)) e y)
. | by ((New:i1 Analyze) THEN (New:i2 Analyze)) THEN Reduce Hyp:-1
. |
. 10. i1 : B
. 11. i2 : B
. 12. (least x:ℕ. (f(x)) e i1) = (least x:ℕ. (f(x)) e i2) ∈ ℕa
. ⊢  i1 = i2  by ApFun: f to: Hyp:12
. |
. 13. f(least x:ℕ. (f(x)) e i1) = f(least x:ℕ. (f(x)) e i2)
. ⊢  i1 = i2  by Rewrite-1 by ∀y:B. f(least x:ℕ. (f(x)) e y) = y ∈ B
. |
. |\
. | 14. f(least x:ℕ. (f(x)) e i1) = f(least x:ℕ. (f(x)) e i2)
. | 15. y : B
. | ⊢  f(least x:ℕ. (f(x)) e y) = y
. |   by BackThru:
. |      Thm*  ∀e:(B→B→ℙ).
. |            IsEqFun(B;e)
. |            ⇒
. |            (∀a:ℕ, f:(ℕa onto→ B), y:B.
. |             f(least x:ℕ. (f(x)) e y) = y)
. |      {nsub_surj_least_preimage_works_gen}
. |
. .\
. . 14. i1 = i2
. . ⊢  i1 = i2  by Hypothesis
END
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