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nsub_surj_least_preimage_works_gen  THM

⊢  ∀B:Type, e:(B→B→𝔹).
|  IsEqFun(B;e)
|  ⇒
|  (∀a:ℕ, f:(ℕa onto→ B), y:B. f(least x:ℕ. (f(x)) e y) = y)  by Auto
|
1. B : Type
2. e : B→B→𝔹
3. IsEqFun(B;e)
4. a : ℕ
5. f : ℕa onto→ B
6. y : B
⊢  f(least x:ℕ. (f(x)) e y) = y
| by ((λx.(f(x)) e y) ∈ {p:(ℕa→𝔹)| ∃i:ℕa. p(i) }  Asserted
|      THEN
|      (Use:[a | p(x):= (f(x)) e y]
|       Inst:
|       Thm*  ∀k:ℕ, p:{p:(ℕk→𝔹)| ∃i:ℕk. p(i) }. p(least i:ℕ. p(i))
|       {least_satisfies}))
|     THEN
|     Analyze5
|
|\
| 5. f : ℕa→B
| 6. Surj(ℕa; B; f)
| 7. y : B
| ⊢  (λx.(f(x)) e y) ∈ {p:(ℕa→𝔹)| ∃i:ℕa. p(i) }
| | by Analyze THEN ReduceSOAps Concl
| |
| ⊢  ∃i:ℕa. (f(i)) e y  by Rewrite by Hyp:3
| |
| ⊢  ∃i:ℕa. f(i) = y  by Witness6: y
| |
| 8. ∃a:ℕa. f(a) = y
| ⊢  ∃i:ℕa. f(i) = y  by SimilarTo: Hyp:-1
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. 5. f : ℕa→B
. 6. Surj(ℕa; B; f)
. 7. y : B
. 8. (λx.(f(x)) e y) ∈ {p:(ℕa→𝔹)| ∃i:ℕa. p(i) }
. 9. (f(least i:ℕ. (f(i)) e y)) e y
. ⊢  f(least x:ℕ. (f(x)) e y) = y  by FwdThru: Hyp:3 on [ Hyp:-1 ]
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. |\
. | ⊢  f(least i:ℕ. (f(i)) e y) ∈ B
. | | by ApFun: f to: least i:ℕ. (f(i)) e y
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. | |\
. | | .....assertion .....
. | | ⊢  (least i:ℕ. (f(i)) e y) ∈ ℕa
. | |   by BackThru:
. | |      Thm*  ∀k:ℕ, p:{p:(ℕk→𝔹)| ∃i:ℕk. p(i) }.
. | |            (least i:ℕ. p(i)) ∈ ℕk
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. | |      {least_wf}
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. | . 10. f(least i:ℕ. (f(i)) e y) ∈ B
. | . ⊢  f(least i:ℕ. (f(i)) e y) ∈ B  by Inclusion -1
. |
. |\
. | ⊢  y ∈ B  by Auto
. |
. .\
. . 10. f(least i:ℕ. (f(i)) e y) = y
. . ⊢  f(least x:ℕ. (f(x)) e y) = y  by Auto
END
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