```
one_one_corr_fams_if_bij_A  THM

⊢  ∀g:(A'→A). Bij(A'; A; g) ⇒ (x:A. B(x)) ∼ (x':A'. B(g(x')))
| by Auto
|
1. A : Type
2. A' : Type
3. B : A→Type
4. g : A'→A
5. Bij(A'; A; g)
⊢  (x:A. B(x)) ∼ (x':A'. B(g(x')))
| by FwdThru:
|    Thm*  ∀f:(A→B). Bij(A; B; f) ⇒ (∃g:(B→A). InvFuns(A;B;f;g))
|    {bij_imp_exists_inv2}
|    on [ Bij(A'; A; g) ]
|    THEN
|    New:f Analyze-1
|
6. f : A→A'
7. InvFuns(A';A;g;f)
⊢  (x:A. B(x)) ∼ (x':A'. B(g(x')))  by Witness: f | g
|
⊢  ∃F:(x:A→B(x)→B(g(f(x)))), G:(x:A→B(g(f(x)))→B(x)).
|  InvFuns(A;A';f;g) & (∀u:A. InvFuns(B(u);B(g(f(u)));F(u);G(u)))
| by Rewrite by ∀y:A. g(f(y)) = y
|
|\
| 8. y : A
| ⊢  g(f(y)) = y  by BackThru: InvFuns(A';A;g;f)
|
.\
. ⊢  ∃F,G:(x:A→B(x)→B(x)).
. |  InvFuns(A;A';f;g) & (∀u:A. InvFuns(B(u);B(u);F(u);G(u)))
. | by Witness: λu.Id | λu.Id THEN Reduce 0
. |
. |\
. | ⊢  InvFuns(A;A';f;g)
. |   by BackThru:
. |      Thm*  InvFuns(A;B;f;g) ⇒ InvFuns(B;A;g;f)
. |      {inv_funs_2_sym}
. |
. .\
. . 8. u : A
. . ⊢  InvFuns(B(u);B(u);Id;Id)
. .   by BackThru: Thm*  InvFuns(A;A;Id;Id)    {inv_funs_2_identity}
END
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