```
one_one_corr_fams_sym  THM

⊢  ∀A,A':Type, B:(A→Type), B':(A'→Type).
|  (x:A. B(x)) ∼ (x':A'. B'(x')) ⇒ (x':A'. B'(x')) ∼ (x:A. B(x))
| by Auto
|
1. A : Type
2. A' : Type
3. B : A→Type
4. B' : A'→Type
5. (x:A. B(x)) ∼ (x':A'. B'(x'))
⊢  (x':A'. B'(x')) ∼ (x:A. B(x))  by (-1) Def THEN ExistHD Hyp:5
|
5. f : A→A'
6. g : A'→A
7. F : x:A→B(x)→B'(f(x))
8. G : x:A→B'(f(x))→B(x)
9. InvFuns(A;A';f;g)
10. ∀u:A. InvFuns(B(u);B'(f(u));F(u);G(u))
⊢  (x':A'. B'(x')) ∼ (x:A. B(x))  by Witness: g | f
|
⊢  ∃F:(x':A'→B'(x')→B(g(x'))), G:(x':A'→B(g(x'))→B'(x')).
|  InvFuns(A';A;g;f) & (∀u:A'. InvFuns(B'(u);B(g(u));F(u);G(u)))
| by G o g ∈ x':A'→B'(x')→B(g(x')) |
|    F o g ∈ x':A'→B(g(x'))→B'(x')
|    Asserted
|
|\
| .....assertion .....
| ⊢  G o g ∈ x':A'→B'(x')→B(g(x'))
| | by Def THEN CBV: x' THEN Analyze
| |
| 11. x' : A'
| ⊢  G(g(x')) ∈ B'(x')→B(g(x'))
|   by ChangeToEqType: B'(f(g(x')))→B(g(x'))
|      THEN
|      Rewrite by f(g(x')) = x' ∈ A' THEN BackThru: Hyp:9
|
|\
| .....assertion .....
| 11. G o g ∈ x':A'→B'(x')→B(g(x'))
| ⊢  F o g ∈ x':A'→B(g(x'))→B'(x')
| | by Def THEN CBV: x' THEN Analyze
| |
| 12. x' : A'
| ⊢  F(g(x')) ∈ B(g(x'))→B'(x')
|   by ChangeToEqType: B(g(x'))→B'(f(g(x')))
|      THEN
|      Rewrite by f(g(x')) = x' ∈ A' THEN BackThru: Hyp:9
|
.\
. 11. G o g ∈ x':A'→B'(x')→B(g(x'))
. 12. F o g ∈ x':A'→B(g(x'))→B'(x')
. ⊢  ∃F:(x':A'→B'(x')→B(g(x'))), G:(x':A'→B(g(x'))→B'(x')).
. |  InvFuns(A';A;g;f) & (∀u:A'. InvFuns(B'(u);B(g(u));F(u);G(u)))
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. | by Witness: G o g | F o g
. |
. |\
. | ⊢  InvFuns(A';A;g;f)
. |   by BackThru:
. |      Thm*  InvFuns(A;B;f;g) ⇒ InvFuns(B;A;g;f)
. |      {inv_funs_2_sym}
. |
. .\
. . 13. u : A'
. . ⊢  InvFuns(B'(u);B(g(u));(G o g)(u);(F o g)(u))
. . | by Witness10: g(u)
. . |
. . 14. InvFuns(B(g(u));B'(f(g(u)));F(g(u));G(g(u)))
. . ⊢  InvFuns(B'(u);B(g(u));(G o g)(u);(F o g)(u))
. . | by Rewrite'-1 by f(g(u)) = u ∈ A'
. . |
. . |\
. . | .....assertion .....
. . | ⊢  f(g(u)) = u  by BackThru: Hyp:9
. . |
. . .\
. . . 14. InvFuns(B(g(u));B'(u);F(g(u));G(g(u)))
. . . ⊢  InvFuns(B'(u);B(g(u));(G o g)(u);(F o g)(u))
. . . | by BackThru:
. . . |    Thm*  InvFuns(A;B;f;g) ⇒ InvFuns(B;A;g;f)
. . . |    {inv_funs_2_sym}
. . . |
. . . ⊢  InvFuns(B(g(u));B'(u);(F o g)(u);(G o g)(u))
. . .   by Reduce Concl
END
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