```
one_one_corr_fams_trans  THM

⊢  ∀A:Type, A',A'':Type, B:(A→Type), B':(A'→Type), B'':(A''→Type).
|  (x:A. B(x)) ∼ (x':A'. B'(x'))
|  ⇒
|  (x':A'. B'(x')) ∼ (x'':A''. B''(x''))
|  ⇒
|  (x:A. B(x)) ∼ (x'':A''. B''(x''))  by Auto
|
1. A : Type
2. A' : Type
3. A'' : Type
4. B : A→Type
5. B' : A'→Type
6. B'' : A''→Type
7. (x:A. B(x)) ∼ (x':A'. B'(x'))
8. (x':A'. B'(x')) ∼ (x'':A''. B''(x''))
⊢  (x:A. B(x)) ∼ (x'':A''. B''(x''))
| by (7) Def THEN ExistHD Hyp:7 THEN (-1) Def THEN ExistHD Hyp:-1
|
7. f : A→A'
8. g : A'→A
9. F : x:A→B(x)→B'(f(x))
10. G : x:A→B'(f(x))→B(x)
11. InvFuns(A;A';f;g)
12. ∀u:A. InvFuns(B(u);B'(f(u));F(u);G(u))
13. f1 : A'→A''
14. g1 : A''→A'
15. F1 : x':A'→B'(x')→B''(f1(x'))
16. G1 : x':A'→B''(f1(x'))→B'(x')
17. InvFuns(A';A'';f1;g1)
18. ∀u:A'. InvFuns(B'(u);B''(f1(u));F1(u);G1(u))
⊢  (x:A. B(x)) ∼ (x'':A''. B''(x''))
| by Witness:
|      f1 o f |
|      g o g1 |
|      λx.F1(f(x)) o F(x) |
|      λx.G(x) o G1(f(x))
|    THEN
|    Reduce Concl
|
|\
| ⊢  InvFuns(A;A'';f1 o f;g o g1)  by Analyze17 THEN Analyze11
| |
| 19. ∀x:A'. g1(f1(x)) = x
| 20. ∀y:A''. f1(g1(y)) = y
| 21. ∀x:A. g(f(x)) = x
| 22. ∀y:A'. f(g(y)) = y
| ⊢  InvFuns(A;A'';f1 o f;g o g1)  by Analyze THEN Reduce Concl
| |
| |\
| | 23. x : A
| | ⊢  g(g1(f1(f(x)))) = x
| |   by Rewrite by Hyp:19 THEN Rewrite by Hyp:21
| |
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| .\
| . 23. y : A''
| . ⊢  f1(f(g(g1(y)))) = y
| .   by Rewrite by Hyp:22 THEN Rewrite by Hyp:20
|
.\
. 19. u : A
. ⊢  InvFuns(B(u);B''(f1(f(u)));F1(f(u)) o F(u);G(u) o G1(f(u)))
. | by Witness12: u THEN Analyze-1
. |    THEN
. |    Witness18: f(u) THEN Analyze-1
. |
. 20. ∀x:B(u). G(u,F(u,x)) = x
. 21. ∀y:B'(f(u)). F(u,G(u,y)) = y
. 22. ∀x:B'(f(u)). G1(f(u),F1(f(u),x)) = x
. 23. ∀y:B''(f1(f(u))). F1(f(u),G1(f(u),y)) = y
. ⊢  InvFuns(B(u);B''(f1(f(u)));F1(f(u)) o F(u);G(u) o G1(f(u)))
. | by Analyze THEN Reduce Concl
. |
. |\
. | 24. x : B(u)
. | ⊢  G(u,G1(f(u),F1(f(u),F(u,x)))) = x
. |   by Rewrite by Hyp:22 THEN Rewrite by Hyp:20
. |
. .\
. . 24. y : B''(f1(f(u)))
. . ⊢  F1(f(u),F(u,G(u,G1(f(u),y)))) = y
. .   by Rewrite by Hyp:21 THEN Rewrite by Hyp:23
END
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