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one_one_corr_is_equiv_rel  THM

⊢  EquivRel X,Y:Type. X ∼ Y
| by Def of X ∼ Y THEN BackThru: equivrel_characterization
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| 1. X : Type
| ⊢  ∃f,g:(X→X). InvFuns(X;X;f;g)  by InvFuns(X;X;Id;Id)
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| 1. X : Type
| 2. Y : Type
| 3. ∃f:(X→Y), g:(Y→X). InvFuns(X;Y;f;g)
| ⊢  ∃g:(Y→X), f:(X→Y). InvFuns(Y;X;g;f)
|   by Thm*  InvFuns(A;B;f;g) ⇒ InvFuns(B;A;g;f)    {inv_funs_2_sym}
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. 1. X : Type
. 2. Y : Type
. 3. ∃f1:(X→Y), g1:(Y→X). InvFuns(X;Y;f1;g1)
. 4. Z : Type
. 5. ∃f2:(Y→Z), g2:(Z→Y). InvFuns(Y;Z;f2;g2)
. ⊢  ∃f:(X→Z), g:(Z→X). InvFuns(X;Z;f;g)
. | by ExistHD Hyp:5 THEN ExistHD Hyp:3
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. 3. f1 : X→Y
. 4. g1 : Y→X
. 5. InvFuns(X;Y;f1;g1)
. 6. Z : Type
. 7. f2 : Y→Z
. 8. g2 : Z→Y
. 9. InvFuns(Y;Z;f2;g2)
. ⊢  ∃f:(X→Z), g:(Z→X). InvFuns(X;Z;f;g)  by Analyze9 THEN Analyze5
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. 10. ∀x:Y. g2(f2(x)) = x
. 11. ∀y:Z. f2(g2(y)) = y
. 12. ∀x:X. g1(f1(x)) = x
. 13. ∀y:Y. f1(g1(y)) = y
. ⊢  ∃f:(X→Z), g:(Z→X). InvFuns(X;Z;f;g)
. | by Witness: f2 o f1 | g1 o g2 THEN Analyze THEN Reduce 0
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. | 14. x : X
. | ⊢  g1(g2(f2(f1(x)))) = x
. |   by Rewrite by Hyp:10 THEN Rewrite by Hyp:12
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. . 14. y : Z
. . ⊢  f2(f1(g1(g2(y)))) = y
. .   by Rewrite by Hyp:13 THEN Rewrite by Hyp:11
END
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