```
partition_type  THM

⊢  ∀P:(A→B→Prop).
|  (∀x:A. ∃!y:B. P(x;y)) ⇒ (A ∼ (y:B×{x:A| P(x;y) }))  by Auto
|
1. A : Type
2. B : Type
3. P : A→B→Prop
4. ∀x:A. ∃!y:B. P(x;y)
⊢  A ∼ (y:B×{x:A| P(x;y) })
| by FwdThru:
|    Thm*  (∀x:A. ∃!y:B(x). P(x;y))
|          ⇒
|          (∃f:(x:A→B(x)). ∀x:A. P(x;f(x)))
|    {fun_description}
|    on [ Hyp:-1 ]
|    THEN
|    Analyze-1
|
5. f : A→B
6. ∀x:A. P(x;f(x))
⊢  A ∼ (y:B×{x:A| P(x;y) })  by Witness: λx.<f(x),x> | λyx.2of(yx)
|
|\
| 7. x : A
| ⊢  x ∈ {x@0:A| P(x@0;f(x)) }
|   by Analyze THEN BackThru: ∀x:A. P(x;f(x))
|
|\
| ⊢  InvFuns(A;y:B×{x:A| P(x;y) };λx.<f(x),x>;λyx.2of(yx))
| | by Analyze THEN New:u Analyze THEN Reduce Concl
| |
| 7. u : y:B×{x:A| P(x;y) }
| ⊢  <f(2of(u)),2of(u)> = u ∈ y:B×{x:A| P(x;y) }  by Analyze-1
| |
| 7. y : B
| 8. u1 : {x:A| P(x;y) }
| ⊢  <f(2of(<y,u1>)),2of(<y,u1>)> = <y,u1> ∈ y:B×{x:A| P(x;y) }
| | by Reduce Concl
| |
| ⊢  <f(u1),u1> = <y,u1> ∈ y:B×{x:A| P(x;y) }  by Witness4: u1
| |
| 9. ∃!y:B. P(u1;y)
| ⊢  <f(u1),u1> = <y,u1> ∈ y:B×{x:A| P(x;y) }  by Analyze-1
| |
| 9. y1 : B
| 10. y1 is the y:B. P(u1;y)
| ⊢  <f(u1),u1> = <y,u1> ∈ y:B×{x:A| P(x;y) }  by Analyze8
| |
| 8. u1 : A
| 9. P(u1;y)
| 10. y1 : B
| 11. y1 is the y:B. P(u1;y)
| ⊢  <f(u1),u1> = <y,u1> ∈ y:B×{x:A| P(x;y) }  by Witness6: u1
| |
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| 12. P(u1;f(u1))
| ⊢  <f(u1),u1> = <y,u1> ∈ y:B×{x:A| P(x;y) }
| | by FwdThru: Hyp:11 on [ P(u1;y) ]
| |    THEN
| |    FwdThru: Hyp:11 on [ P(u1;f(u1)) ]
| |
| 13. y = y1
| 14. f(u1) = y1
| ⊢  <f(u1),u1> = <y,u1> ∈ y:B×{x:A| P(x;y) }  by Analyze
|
.\
. 7. (y:B×{x:A| P(x;y) })→A
. 8. x : A
. ⊢  x ∈ {x@0:A| P(x@0;f(x)) }
.   by Analyze THEN BackThru: ∀x:A. P(x;f(x))
END
```