```
pigeon_hole  THM

⊢  ∀m,k:ℕ, f:(ℕm→ℕk). k<m ⇒ (∃x,y:ℕm. x ≠ y & f(x) = f(y))  by Auto
|
1. m : ℕ
2. k : ℕ
3. f : ℕm→ℕk
4. k<m
⊢  ∃x,y:ℕm. x ≠ y & f(x) = f(y)
| by Inst:
|    Thm*  ∀m:ℕ, f:(ℕm→ℤ).
|          ¬Inj(ℕm; ℤ; f) ⇒ (∃x:ℕm, y:ℕx. f(x) = f(y))
|    {finite_inj_counter_example}
|    on
|    [m | f]
|
|\
| .....antecedent .....
| ⊢  ¬Inj(ℕm; ℤ; f)  by ¬m≤k  Asserted
| |
| 5. ¬m≤k
| ⊢  ¬Inj(ℕm; ℤ; f)  by SimilarTo -1
| |
| 5. Inj(ℕm; ℤ; f)
| ⊢  m≤k
| | by BackThru:
| |    Thm*  (∃f:(ℕm→ℕk). Inj(ℕm; ℕk; f)) ⇒ m≤k    {inj_imp_le}
| |
| ⊢  ∃f:(ℕm→ℕk). Inj(ℕm; ℕk; f)  by Witness: f
| |
| ⊢  Inj(ℕm; ℕk; f)  by SimilarTo: Hyp:5
|
.\
. 5. ∃x:ℕm, y:ℕx. f(x) = f(y)
. ⊢  ∃x,y:ℕm. x ≠ y & f(x) = f(y)  by SimilarTo: -1
END
```