```
simple_card_cross_assoc  THM

⊢  (A×B×C) ∼ ((A×B)×C)  by UnivCD
|
1. A : Type
2. B : Type
3. C : Type
⊢  (A×B×C) ∼ ((A×B)×C)
| by Witness:
|      λabc.abc/a,bc. bc/b,c. <<a,b>,c> |
|      λxyz.xyz/xy,z. xy/x,y. <x,y,z>
|
⊢  InvFuns(A×B×C;(A×B)×C
|         ;λabc.abc/a,bc. bc/b,c. <<a,b>,c>
|         ;λxyz.xyz/xy,z. xy/x,y. <x,y,z>)
| by Analyze THENL [New:abc Analyze;New:xyz Analyze]
|
|\
| 4. abc : A×B×C
| ⊢  (λxyz.xyz/xy,z. xy/x,y. <x,y,z>)
| |  ((λabc.abc/a,bc. bc/b,c. <<a,b>,c>)(abc))
| |  =
| |  abc  by Analyze-1
| |
| 4. a1 : A
| 5. a2 : B×C
| ⊢  (λxyz.xyz/xy,z. xy/x,y. <x,y,z>)
| |  ((λabc.abc/a,bc. bc/b,c. <<a,b>,c>)(<a1,a2>))
| |  =
| |  <a1,a2>  by Analyze-1
| |
| 5. a3 : B
| 6. a4 : C
| ⊢  (λxyz.xyz/xy,z. xy/x,y. <x,y,z>)
| |  ((λabc.abc/a,bc. bc/b,c. <<a,b>,c>)(<a1,a3,a4>))
| |  =
| |  <a1,a3,a4>  by Reduce Concl
| |
| ⊢  <a1,a3,a4> = <a1,a3,a4>  by Auto
|
.\
. 4. xyz : (A×B)×C
. ⊢  (λabc.abc/a,bc. bc/b,c. <<a,b>,c>)
. |  ((λxyz.xyz/xy,z. xy/x,y. <x,y,z>)(xyz))
. |  =
. |  xyz  by Analyze-1
. |
. 4. x1 : A×B
. 5. x2 : C
. ⊢  (λabc.abc/a,bc. bc/b,c. <<a,b>,c>)
. |  ((λxyz.xyz/xy,z. xy/x,y. <x,y,z>)(<x1,x2>))
. |  =
. |  <x1,x2>  by Analyze-2
. |
. 4. x3 : A
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. 5. x4 : B
. 6. x2 : C
. ⊢  (λabc.abc/a,bc. bc/b,c. <<a,b>,c>)
. |  ((λxyz.xyz/xy,z. xy/x,y. <x,y,z>)(<<x3,x4>,x2>))
. |  =
. |  <<x3,x4>,x2>  by Reduce Concl
. |
. ⊢  <<x3,x4>,x2> = <<x3,x4>,x2>  by Auto
END
```