```
surjection_type_functionality_wrt_ooc  THM

⊢  (A ∼ A') ⇒ (B ∼ B') ⇒ ((A onto→ B) ∼ (A' onto→ B'))  by Auto
|
1. A : Type
2. A' : Type
3. B : Type
4. B' : Type
5. A ∼ A'
6. B ∼ B'
⊢  (A onto→ B) ∼ (A' onto→ B')
| by Analyze A ∼ A' and B ∼ B' and extract facts about cancellation
|    and surjection.
|
5. f1 : A→A'
6. g1 : A'→A
7. InvFuns(A;A';f1;g1)
8. f : B→B'
9. g : B'→B
10. InvFuns(B;B';f;g)
11. ∀x:B. g(f(x)) = x
12. ∀y:B'. f(g(y)) = y
13. ∀x:A. g1(f1(x)) = x
14. ∀y:A'. f1(g1(y)) = y
15. Surj(A; A'; f1)
16. Surj(A'; A; g1)
17. Surj(B; B'; f)
18. Surj(B'; B; g)
⊢  (A onto→ B) ∼ (A' onto→ B')
| by Witness: λh.f o h o g1 with type (A onto→ B)→A' onto→ B'
|    THEN
|    Witness: λh.g o h o f1 with type (A' onto→ B')→A onto→ B
|
|\
| .....assertion .....
| ⊢  (λh.f o h o g1) ∈ (A onto→ B)→A' onto→ B'  by Auto
| |
| 19. h : A onto→ B
| ⊢  f o h o g1 ∈ A' onto→ B'  by Analyze-1 THEN Analyze
| |
| .....set predicate .....
| 19. h : A→B
| 20. Surj(A; B; h)
| ⊢  Surj(A'; B'; f o h o g1)
|   by Repeat
|      BackThru:
|      Thm*  Surj(A; B; g) ⇒ Surj(B; C; f) ⇒ Surj(A; C; f o g)
|      {comp_preserves_surj}
|
|\
| .....assertion .....
| 19. (λh.f o h o g1) ∈ (A onto→ B)→A' onto→ B'
| ⊢  (λh.g o h o f1) ∈ (A' onto→ B')→A onto→ B
|   by Same as other case.
|
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. 19. (λh.f o h o g1) ∈ (A onto→ B)→A' onto→ B'
. 20. (λh.g o h o f1) ∈ (A' onto→ B')→A onto→ B
. ⊢  InvFuns(A onto→ B;A' onto→ B';λh.f o h o g1;λh.g o h o f1)
. | by Analyze THEN Reduce Concl
. |
. |\
. | 21. x : A onto→ B
. | ⊢  g o f o x o g1 o f1 = x  by New:e Analyze-1
. | |
. | 21. e : A→B
. | 22. Surj(A; B; e)
. | ⊢  g o f o e o g1 o f1 = e ∈ A onto→ B  by Analyze
. | |
. | |\
. | | ⊢  g o f o e o g1 o f1 = e
. | | | by FunExtensionality THEN Reduce Concl
. | | |
. | | 23. x : A
. | | ⊢  g(f(e(g1(f1(x))))) = e(x)
. | |   by Repeat Rewrite by Hyp:11 | Hyp:12 | Hyp:13 | Hyp:14
. | |
. | .\
. | . .....set predicate .....
. | . ⊢  Surj(A; B; g o f o e o g1 o f1)
. | .   by Repeat
. | .      BackThru:
. | .      Thm*  Surj(A; B; g) ⇒ Surj(B; C; f) ⇒ Surj(A; C; f o g)
. | .      {comp_preserves_surj}
. |
. .\
. . 21. y : A' onto→ B'
. . ⊢  f o g o y o f1 o g1 = y  by Same as other case.
END
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