```
auto2_lemma_0  THM

⊢  ∀P:(T→Prop).
|  (∀x:T. Dec(P(x))) & Dec(∃x:T. ¬P(x)) ⇒ Dec(∀x:T. P(x))
| by GenUnivCD
|
1. T : Type
2. P : T→Prop
3. (∀x:T. Dec(P(x))) & Dec(∃x:T. ¬P(x))
⊢  Dec(∀x:T. P(x))  by Analyze 3
|
3. ∀x:T. Dec(P(x))
4. Dec(∃x:T. ¬P(x))
⊢   by Unfold ‘decidable‘ 0 THEN Unfold ‘decidable‘ 4
|
4. (∃x:T. ¬P(x)) ∨ ¬(∃x:T. ¬P(x))
⊢  (∀x:T. P(x)) ∨ ¬(∀x:T. P(x))  by Analyze 4
|
|\
| 4. ∃x:T. ¬P(x)
| ⊢   by Sel 2 (Analyze 0)
| |
| ⊢  ¬(∀x:T. P(x))  by Analyze 4
| |
| 4. x : T
| 5. ¬P(x)
| ⊢   by Analyze 0
| |
| 6. ∀x:T. P(x)
| ⊢  False  by Witness6 x
|
.\
. 4. ¬(∃x:T. ¬P(x))
. ⊢   by Sel 1 (Analyze 0)
. |
. ⊢  ∀x:T. P(x)  by Analyze 0
. |
. 5. x : T
. ⊢  P(x)  by Witness3 x
. |
. 3. ¬(∃x:T. ¬P(x))
. 4. x : T
. 5. Dec(P(x))
. ⊢   by Unfold ‘decidable‘ 5
. |
. 5. P(x) ∨ ¬P(x)
. ⊢   by Analyze 5
. |
. |\
. | 5. P(x)
. | ⊢   by Auto
. |
. .\
. . 5. ¬P(x)
. . ⊢   by Analyze 3
```

```
. . |
. . 3. x : T
. . 4. ¬P(x)
. . ⊢  ∃x:T. ¬P(x)  by Witness x
END
```