```
auto2_lemma_1  THM

⊢  ∀P,Q:(T→Prop).
|  (∀t:T. P(t) ⟺ Q(t)) ⇒ ({t:T| P(t) } ∼ {t:T| Q(t) })  by UnivCD
|
1. T : Type
2. P : T→Prop
3. Q : T→Prop
4. ∀t:T. P(t) ⟺ Q(t)
⊢  {t:T| P(t) } ∼ {t:T| Q(t) }  by Unfold ‘one_one_corr‘ 0
|
⊢  ∃f:({t:T| P(t) }→{t:T| Q(t) }), g:({t:T| Q(t) }→{t:T| P(t) }).
|  InvFuns({t:T| P(t) }; {t:T| Q(t) }; f; g)  by Witness Id
|
|\
| .....wf .....
| ⊢  Id ∈ {t:T| P(t) }→{t:T| Q(t) }  by Unfold ‘identity‘ 0
| |
| ⊢  (λx.x) ∈ {t:T| P(t) }→{t:T| Q(t) }  by Analyze
| |
| 5. x : {t:T| P(t) }
| ⊢  x ∈ {t:T| Q(t) }  by Analyze 5
| |
| 5. x : T
| 6. P(x)
| ⊢   by Analyze THEN Try Auto
| |
| .....set predicate .....
| ⊢  Q(x)  by Witness4 x
| |
| 4. x : T
| 5. P(x)
| 6. P(x) ⇒ Q(x)
| 7. (P(x)) ⇐ (Q(x))
| ⊢   by Analyze 6
|
.\
. ⊢  ∃g:({t:T| Q(t) }→{t:T| P(t) }).
. |  InvFuns({t:T| P(t) }; {t:T| Q(t) }; Id; g)  by Witness Id
. |
. |\
. | .....wf .....
. | ⊢  Id ∈ {t:T| Q(t) }→{t:T| P(t) }
. | | by Unfold ‘identity‘ 0 THEN Analyze
. | |
. | 5. x : {t:T| Q(t) }
. | ⊢  x ∈ {t:T| P(t) }  by Analyze 5
. | |
. | 5. x : T
. | 6. Q(x)
. | ⊢   by Witness4 x
. | |
. | 4. x : T
. | 5. Q(x)
. | 6. P(x) ⟺ Q(x)
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. | ⊢   by Analyze 6
. | |
. | 6. P(x) ⇒ Q(x)
. | 7. (P(x)) ⇐ (Q(x))
. | ⊢   by Analyze 7
. | |
. | 7. P(x)
. | ⊢   by Analyze THEN Try Auto
. |
. |\
. | ⊢  InvFuns({t:T| P(t) }; {t:T| Q(t) }; Id; Id)
. | | by Unfold ‘inv_funs‘ 0
. | |
. | ⊢  Id o Id = Id & Id o Id = Id  by Unfold ‘identity‘ 0
. | |
. | ⊢  (λx.x) o (λx.x) = Id & (λx.x) o (λx.x) = Id
. | | by Unfold ‘compose‘ 0 THEN Reduce 0
. | |
. | ⊢  (λx.x) = Id & (λx.x) = Id  by Fold ‘identity‘ 0
. | |
. | ⊢  Id = Id & Id = Id  by Unfold ‘tidentity‘ 0
. | |
. | ⊢  Id = Id & Id = Id  by Analyze 0
. |
. .\
. . 5. {t:T| Q(t) }→{t:T| P(t) }
. . ⊢  Id ∈ {t:T| P(t) }→{t:T| Q(t) }  by Unfold ‘identity‘ 0
. . |
. . ⊢  (λx.x) ∈ {t:T| P(t) }→{t:T| Q(t) }  by Analyze
. . |
. . 6. x : {t:T| P(t) }
. . ⊢  x ∈ {t:T| Q(t) }  by Analyze 6
. . |
. . 6. x : T
. . 7. P(x)
. . ⊢   by Analyze THEN Try Auto
. . |
. . .....set predicate .....
. . ⊢  Q(x)  by Witness4 x
. . |
. . 4. {t:T| Q(t) }→{t:T| P(t) }
. . 5. x : T
. . 6. P(x)
. . 7. P(x) ⟺ Q(x)
. . ⊢   by Analyze 7 THEN Analyze 7
END
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