auto2_lemma_6 THM

```
⊢  ∀R:(T→Prop). Fin(T) & (∀t:T. Dec(R(t))) ⇒ Dec(∃t:T. R(t))
| by GenUnivCD
|
1. T : Type
2. R : T→Prop
3. Fin(T) & (∀t:T. Dec(R(t)))
⊢  Dec(∃t:T. R(t))  by Analyze 3
|
3. Fin(T)
4. ∀t:T. Dec(R(t))
⊢
| by Assert
|    (∀n:ℕ, T:Type, R:(T→Prop).
|     (∃f:(ℕn→T). Bij(ℕn; T; f)) & (∀t:T. Dec(R(t))) ⇒
|     Dec(∃t:T. R(t)))
|
|\
| .....assertion .....
| ⊢  ∀n:ℕ, T:Type, R:(T→Prop).
| |  (∃f:(ℕn→T). Bij(ℕn; T; f)) & (∀t:T. Dec(R(t))) ⇒
| |  Dec(∃t:T. R(t))  by Thin 4 THEN Thin 3 THEN Thin 2 THEN Thin 1
| |
| (no hyps)
| ⊢  ∀n:ℕ, T:Type, R:(T→Prop).
| |  (∃f:(ℕn→T). Bij(ℕn; T; f)) & (∀t:T. Dec(R(t))) ⇒
| |  Dec(∃t:T. R(t))  by Analyze 0
| |
| 1. n : ℕ
| ⊢  ∀T:Type, R:(T→Prop).
| |  (∃f:(ℕn→T). Bij(ℕn; T; f)) & (∀t:T. Dec(R(t))) ⇒
| |  Dec(∃t:T. R(t))  by NatInd 1
| |
| |\
| | .....basecase .....
| | (no hyps)
| | ⊢  ∀T:Type, R:(T→Prop).
| | |  (∃f:(ℕ0→T). Bij(ℕ0; T; f)) & (∀t:T. Dec(R(t))) ⇒
| | |  Dec(∃t:T. R(t))  by UnivCD
| | |
| | 1. T : Type
| | 2. R : T→Prop
| | 3. (∃f:(ℕ0→T). Bij(ℕ0; T; f)) & (∀t:T. Dec(R(t)))
| | ⊢  Dec(∃t:T. R(t))  by Analyze 3
| | |
| | 3. ∃f:(ℕ0→T). Bij(ℕ0; T; f)
| | 4. ∀t:T. Dec(R(t))
| | ⊢   by Unfold ‘decidable‘ 0
| | |
| | ⊢  (∃t:T. R(t)) ∨ ¬(∃t:T. R(t))  by Sel 2 (Analyze 0)
| | |
| | ⊢  ¬(∃t:T. R(t))  by Analyze 0
| | |
| | 5. ∃t:T. R(t)
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| | ⊢  False  by Analyze 5
| | |
| | 5. t : T
| | 6. R(t)
| | ⊢   by Analyze 3
| | |
| | 3. f : ℕ0→T
| | 4. Bij(ℕ0; T; f)
| | 5. ∀t:T. Dec(R(t))
| | 6. t : T
| | 7. R(t)
| | ⊢   by Analyze 4
| | |
| | 4. Inj(ℕ0; T; f)
| | 5. Surj(ℕ0; T; f)
| | 6. ∀t:T. Dec(R(t))
| | 7. t : T
| | 8. R(t)
| | ⊢   by Unfold ‘surject‘ 5
| | |
| | 5. ∀b:T. ∃a:ℕ0. f(a) = b
| | 6. ∀t:T. Dec(R(t))
| | 7. t : T
| | 8. R(t)
| | ⊢   by Witness5 t
| | |
| | 5. ∀t:T. Dec(R(t))
| | 6. t : T
| | 7. R(t)
| | 8. ∃a:ℕ0. f(a) = t
| | ⊢   by Analyze 8
| |
| .\
| . .....upcase .....
| . 1. n : ℤ
| . 2. 0<n
| . 3. ∀T:Type, R:(T→Prop).
| . |  (∃f:(ℕ(n-1)→T). Bij(ℕ(n-1); T; f)) & (∀t:T. Dec(R(t))) ⇒
| . |  Dec(∃t:T. R(t))
| . ⊢   by GenUnivCD
| . |
| . 4. T : Type
| . 5. R : T→Prop
| . 6. (∃f:(ℕn→T). Bij(ℕn; T; f)) & (∀t:T. Dec(R(t)))
| . ⊢  Dec(∃t:T. R(t))  by Analyze 6
| . |
| . 6. ∃f:(ℕn→T). Bij(ℕn; T; f)
| . 7. ∀t:T. Dec(R(t))
| . ⊢   by Analyze 6
| . |
| . 6. f : ℕn→T
| . 7. Bij(ℕn; T; f)
| . 8. ∀t:T. Dec(R(t))
| . ⊢   by Witness3 {t:T| ¬t = f(n-1) }
| . |
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| . 3. T : Type
| . 4. R : T→Prop
| . 5. f : ℕn→T
| . 6. Bij(ℕn; T; f)
| . 7. ∀t:T. Dec(R(t))
| . 8. ∀R:({t:T| ¬t = f(n-1) }→Prop).
| . |  (∃f@0:(ℕ(n-1)→{t:T| ¬t = f(n-1) }).
| . |   Bij(ℕ(n-1); {t:T| ¬t = f(n-1) }; f@0))
| . |  & (∀t:{t:T| ¬t = f(n-1) }. Dec(R(t)))
| . |  ⇒
| . |  Dec(∃t:{t:T| ¬t = f(n-1) }. R(t))
| . ⊢   by Witness8 R
| . |
| . 8. (∃f@0:(ℕ(n-1)→{t:T| ¬t = f(n-1) }).
| . |   Bij(ℕ(n-1); {t:T| ¬t = f(n-1) }; f@0))
| . |  & (∀t:{t:T| ¬t = f(n-1) }. Dec(R(t)))
| . |  ⇒
| . |  Dec(∃t:{t:T| ¬t = f(n-1) }. R(t))
| . ⊢   by Analyze 8
| . |
| . |\
| . | .....antecedent .....
| . | 7. ∀t:T. Dec(R(t))
| . | ⊢  (∃f@0:(ℕ(n-1)→{t:T| ¬t = f(n-1) }).
| . | |   Bij(ℕ(n-1); {t:T| ¬t = f(n-1) }; f@0))
| . | |  & (∀t:{t:T| ¬t = f(n-1) }. Dec(R(t)))  by Analyze 0
| . | |
| . | |\
| . | | ⊢  ∃f@0:(ℕ(n-1)→{t:T| ¬t = f(n-1) }).
| . | | |  Bij(ℕ(n-1); {t:T| ¬t = f(n-1) }; f@0)  by Witness f
| . | | |
| . | | |\
| . | | | .....wf .....
| . | | | ⊢  f ∈ ℕ(n-1)→{t:T| ¬t = f(n-1) }  by ExtWith [] [ℕn→T]
| . | | | |
| . | | | 8. x : ℕ(n-1)
| . | | | ⊢  f(x) ∈ {t:T| ¬t = f(n-1) }  by Analyze
| . | | | |
| . | | | |\
| . | | | | ⊢  f(x) ∈ T  by Auto
| . | | | |
| . | | | .\
| . | | | . .....set predicate .....
| . | | | . ⊢  ¬f(x) = f(n-1)  by Analyze 0
| . | | | . |
| . | | | . 9. f(x) = f(n-1)
| . | | | . ⊢  False  by Unfold ‘biject‘ 6
| . | | | . |
| . | | | . 6. Inj(ℕn; T; f) & Surj(ℕn; T; f)
| . | | | . 7. ∀t:T. Dec(R(t))
| . | | | . 8. x : ℕ(n-1)
| . | | | . 9. f(x) = f(n-1)
| . | | | . ⊢   by Analyze 6
| . | | | . |
| . | | | . 6. Inj(ℕn; T; f)
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| . | | | . 7. Surj(ℕn; T; f)
| . | | | . 8. ∀t:T. Dec(R(t))
| . | | | . 9. x : ℕ(n-1)
| . | | | . 10. f(x) = f(n-1)
| . | | | . ⊢   by Unfold ‘inject‘ 6
| . | | | . |
| . | | | . 6. ∀a1,a2:ℕn. f(a1) = f(a2) ⇒ a1 = a2
| . | | | . 7. Surj(ℕn; T; f)
| . | | | . 8. ∀t:T. Dec(R(t))
| . | | | . 9. x : ℕ(n-1)
| . | | | . 10. f(x) = f(n-1)
| . | | | . ⊢   by Witness6 x
| . | | | . |
| . | | | . 6. Surj(ℕn; T; f)
| . | | | . 7. ∀t:T. Dec(R(t))
| . | | | . 8. x : ℕ(n-1)
| . | | | . 9. f(x) = f(n-1)
| . | | | . 10. ∀a2:ℕn. f(x) = f(a2) ⇒ x = a2
| . | | | . ⊢   by Witness10 n-1
| . | | | . |
| . | | | . 10. f(x) = f(n-1) ⇒ x = n-1
| . | | | . ⊢   by Analyze 10
| . | | |
| . | | |\
| . | | | ⊢  Bij(ℕ(n-1); {t:T| ¬t = f(n-1) }; f)  by Analyze 0
| . | | | |
| . | | | |\
| . | | | | ⊢  Inj(ℕ(n-1); {t:T| ¬t = f(n-1) }; f)
| . | | | | | by Unfold ‘inject‘ 0
| . | | | | |
| . | | | | ⊢  ∀a1,a2:ℕ(n-1). f(a1) = f(a2) ⇒ a1 = a2  by UnivCD
| . | | | | |
| . | | | | |\
| . | | | | | 8. a1 : ℕ(n-1)
| . | | | | | 9. a2 : ℕ(n-1)
| . | | | | | 10. f(a1) = f(a2)
| . | | | | | ⊢  a1 = a2  by Analyze 6
| . | | | | | |
| . | | | | | 6. Inj(ℕn; T; f)
| . | | | | | 7. Surj(ℕn; T; f)
| . | | | | | 8. ∀t:T. Dec(R(t))
| . | | | | | 9. a1 : ℕ(n-1)
| . | | | | | 10. a2 : ℕ(n-1)
| . | | | | | 11. f(a1) = f(a2)
| . | | | | | ⊢   by Unfold ‘inject‘ 6
| . | | | | | |
| . | | | | | 6. ∀a1,a2:ℕn. f(a1) = f(a2) ⇒ a1 = a2
| . | | | | | 7. Surj(ℕn; T; f)
| . | | | | | 8. ∀t:T. Dec(R(t))
| . | | | | | 9. a1 : ℕ(n-1)
| . | | | | | 10. a2 : ℕ(n-1)
| . | | | | | 11. f(a1) = f(a2)
| . | | | | | ⊢   by InstHyp [a1;a2] 6 THEN Try Auto
| . | | | | | |
| . | | | | | .....antecedent .....
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| . | | | | | | ⊢  f(a1) = f(a2)  by Analyze 11
| . | | | | | |
| . | | | | | |\
| . | | | | | | 8. a1 : ℕ(n-1)
| . | | | | | | 9. ℕ(n-1)
| . | | | | | | ⊢  f(a1) ∈ {t:T| ¬t = f(n-1) }
| . | | | | | | | by Analyze THEN Try Auto
| . | | | | | | |
| . | | | | | | .....set predicate .....
| . | | | | | | ⊢  ¬f(a1) = f(n-1)  by Analyze 0
| . | | | | | | |
| . | | | | | | 10. f(a1) = f(n-1)
| . | | | | | | ⊢  False  by Analyze 6
| . | | | | | | |
| . | | | | | | 6. Inj(ℕn; T; f)
| . | | | | | | 7. Surj(ℕn; T; f)
| . | | | | | | 8. ∀t:T. Dec(R(t))
| . | | | | | | 9. a1 : ℕ(n-1)
| . | | | | | | 10. ℕ(n-1)
| . | | | | | | 11. f(a1) = f(n-1)
| . | | | | | | ⊢   by Unfold ‘inject‘ 6
| . | | | | | | |
| . | | | | | | 6. ∀a1,a2:ℕn. f(a1) = f(a2) ⇒ a1 = a2
| . | | | | | | 7. Surj(ℕn; T; f)
| . | | | | | | 8. ∀t:T. Dec(R(t))
| . | | | | | | 9. a1 : ℕ(n-1)
| . | | | | | | 10. ℕ(n-1)
| . | | | | | | 11. f(a1) = f(n-1)
| . | | | | | | ⊢   by InstHyp [a1;n-1] 6 THEN Try Auto
| . | | | | | |
| . | | | | | .\
| . | | | | | . 8. ℕ(n-1)
| . | | | | | . 9. a2 : ℕ(n-1)
| . | | | | | . ⊢  f(a2) ∈ {t:T| ¬t = f(n-1) }
| . | | | | | . | by Analyze THEN Try Auto
| . | | | | | . |
| . | | | | | . .....set predicate .....
| . | | | | | . ⊢  ¬f(a2) = f(n-1)  by Analyze 0
| . | | | | | . |
| . | | | | | . 10. f(a2) = f(n-1)
| . | | | | | . ⊢  False  by Analyze 6
| . | | | | | . |
| . | | | | | . 6. Inj(ℕn; T; f)
| . | | | | | . 7. Surj(ℕn; T; f)
| . | | | | | . 8. ∀t:T. Dec(R(t))
| . | | | | | . 9. ℕ(n-1)
| . | | | | | . 10. a2 : ℕ(n-1)
| . | | | | | . 11. f(a2) = f(n-1)
| . | | | | | . ⊢   by Unfold ‘inject‘ 6
| . | | | | | . |
| . | | | | | . 6. ∀a1,a2:ℕn. f(a1) = f(a2) ⇒ a1 = a2
| . | | | | | . 7. Surj(ℕn; T; f)
| . | | | | | . 8. ∀t:T. Dec(R(t))
| . | | | | | . 9. ℕ(n-1)
| . | | | | | . 10. a2 : ℕ(n-1)
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| . | | | | . 11. f(a2) = f(n-1)
| . | | | | . ⊢   by InstHyp [a2;n-1] 6 THEN Try Auto
| . | | | |
| . | | | .\
| . | | | . ⊢  Surj(ℕ(n-1); {t:T| ¬t = f(n-1) }; f)
| . | | | . | by Unfold ‘surject‘ 0
| . | | | . |
| . | | | . ⊢  ∀b:{t:T| ¬t = f(n-1) }. ∃a:ℕ(n-1). f(a) = b
| . | | | . | by Analyze 0
| . | | | . |
| . | | | . 8. b : {t:T| ¬t = f(n-1) }
| . | | | . ⊢  ∃a:ℕ(n-1). f(a) = b  by Analyze 8
| . | | | . |
| . | | | . 8. b : T
| . | | | . 9. ¬b = f(n-1)  [not for witness]
| . | | | . ⊢   by Analyze 6
| . | | | . |
| . | | | . 6. Inj(ℕn; T; f)
| . | | | . 7. Surj(ℕn; T; f)
| . | | | . 8. ∀t:T. Dec(R(t))
| . | | | . 9. b : T
| . | | | . 10. ¬b = f(n-1)  [not for witness]
| . | | | . ⊢   by Unfold ‘surject‘ 7
| . | | | . |
| . | | | . 7. ∀b:T. ∃a:ℕn. f(a) = b
| . | | | . 8. ∀t:T. Dec(R(t))
| . | | | . 9. b : T
| . | | | . 10. ¬b = f(n-1)  [not for witness]
| . | | | . ⊢   by Witness7 b
| . | | | . |
| . | | | . 7. ∀t:T. Dec(R(t))
| . | | | . 8. b : T
| . | | | . 9. ¬b = f(n-1)  [not for witness]
| . | | | . 10. ∃a:ℕn. f(a) = b
| . | | | . ⊢   by Analyze 10
| . | | | . |
| . | | | . 10. a : ℕn
| . | | | . 11. f(a) = b
| . | | | . ⊢   by Witness a
| . | | | . |
| . | | | . |\
| . | | | . | .....wf .....
| . | | | . | 9. ¬b = f(n-1)
| . | | | . | 10. a : ℕn
| . | | | . | 11. f(a) = b
| . | | | . | ⊢  a ∈ ℕ(n-1)  by Decide (a = n-1)
| . | | | . | |
| . | | | . | |\
| . | | | . | | 12. a = n-1
| . | | | . | | ⊢   by RWH (HypC 12) 11
| . | | | . | | |
| . | | | . | | 11. f(n-1) = b
| . | | | . | | 12. a = n-1
| . | | | . | | ⊢   by SwapEquands 11
| . | | | . | |
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| . | | | . | .\
| . | | | . | . 12. ¬a = n-1
| . | | | . | . ⊢   by Auto
| . | | | . |
| . | | | . |\
| . | | | . | 9. ¬b = f(n-1)
| . | | | . | 10. a : ℕn
| . | | | . | 11. f(a) = b
| . | | | . | ⊢  f(a) = b  by Analyze THEN Try Auto
| . | | | . | |
| . | | | . | .....set predicate .....
| . | | | . | ⊢  ¬f(a) = f(n-1)  by RWH (HypC 11) 0
| . | | | . |
| . | | | . .\
| . | | | . . .....wf .....
| . | | | . . 9. ¬b = f(n-1)
| . | | | . . 10. a : ℕn
| . | | | . . 11. f(a) = b
| . | | | . . 12. a1 : ℕ(n-1)
| . | | | . . ⊢  f(a1) = b ∈ Prop  by Auto
| . | | | . . |
| . | | | . . ⊢  f(a1) ∈ {t:T| ¬t = f(n-1) }
| . | | | . . | by Analyze THEN Try Auto
| . | | | . . |
| . | | | . . .....set predicate .....
| . | | | . . ⊢  ¬f(a1) = f(n-1)  by Analyze 0
| . | | | . . |
| . | | | . . 13. f(a1) = f(n-1)
| . | | | . . ⊢  False  by Unfold ‘inject‘ 6
| . | | | . . |
| . | | | . . 6. ∀a1,a2:ℕn. f(a1) = f(a2) ⇒ a1 = a2
| . | | | . . 7. ∀t:T. Dec(R(t))
| . | | | . . 8. b : T
| . | | | . . 9. ¬b = f(n-1)
| . | | | . . 10. a : ℕn
| . | | | . . 11. f(a) = b
| . | | | . . 12. a1 : ℕ(n-1)
| . | | | . . 13. f(a1) = f(n-1)
| . | | | . . ⊢   by InstHyp [a1;n-1] 6 THEN Try Auto
| . | | |
| . | | | .\
| . | | | . .....wf .....
| . | | | . 8. f@0 : ℕ(n-1)→{t:T| ¬t = f(n-1) }
| . | | | . ⊢  Bij(ℕ(n-1); {t:T| ¬t = f(n-1) }; f@0) ∈ Prop  by Auto
| . | |
| . | | .\
| . | | . ⊢  ∀t:{t:T| ¬t = f(n-1) }. Dec(R(t))  by Analyze 0
| . | | . |
| . | | . 8. t : {t:T| ¬t = f(n-1) }
| . | | . ⊢  Dec(R(t))  by Witness7 t THEN Try Auto
| . |
| . .\
| . . 8. Dec(∃t:{t:T| ¬t = f(n-1) }. R(t))
| . . ⊢   by Analyze 8
| . . |
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| . . |\
| . . | 8. ∃t:{t:T| ¬t = f(n-1) }. R(t)
| . . | ⊢   by Unfold ‘decidable‘ 0
| . . | |
| . . | ⊢  (∃t:T. R(t)) ∨ ¬(∃t:T. R(t))  by Sel 1 (Analyze 0)
| . . | |
| . . | ⊢  ∃t:T. R(t)  by Analyze 8
| . . | |
| . . | 8. t : {t:T| ¬t = f(n-1) }
| . . | 9. R(t)
| . . | ⊢   by Analyze 8
| . . | |
| . . | 8. t : T
| . . | 9. ¬t = f(n-1)  [not for witness]
| . . | 10. R(t)
| . . | ⊢   by Witness t
| . . |

| . . .\
| . . . 8. ¬(∃t:{t:T| ¬t = f(n-1) }. R(t))
| . . . ⊢   by Witness7 f(n-1)
| . . . |
| . . . 7. ¬(∃t:{t:T| ¬t = f(n-1) }. R(t))
| . . . 8. Dec(R(f(n-1)))
| . . . ⊢   by Analyze 8
| . . . |
| . . . |\
| . . . | 8. R(f(n-1))
| . . . | ⊢   by Unfold ‘decidable‘ 0
| . . . | |
| . . . | ⊢  (∃t:T. R(t)) ∨ ¬(∃t:T. R(t))  by Sel 1 (Analyze 0)
| . . . | |
| . . . | ⊢  ∃t:T. R(t)  by Witness f(n-1) THEN Try Auto
| . . . |
| . . . .\
| . . . . 8. ¬R(f(n-1))
| . . . . ⊢   by Unfold ‘decidable‘ 0
| . . . . |
| . . . . ⊢  (∃t:T. R(t)) ∨ ¬(∃t:T. R(t))  by Sel 2 (Analyze 0)
| . . . . |
| . . . . ⊢  ¬(∃t:T. R(t))  by Analyze 0
| . . . . |
| . . . . 9. ∃t:T. R(t)
| . . . . ⊢  False  by Analyze 9
| . . . . |
| . . . . 9. t : T
| . . . . 10. R(t)
| . . . . ⊢   by Analyze 6
| . . . . |
| . . . . 6. Inj(ℕn; T; f)
| . . . . 7. Surj(ℕn; T; f)
| . . . . 8. ¬(∃t:{t:T| ¬t = f(n-1) }. R(t))
| . . . . 9. ¬R(f(n-1))
| . . . . 10. t : T
| . . . . 11. R(t)
| . . . . ⊢   by Unfold ‘surject‘ 7
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| . . . . |
| . . . . 7. ∀b:T. ∃a:ℕn. f(a) = b
| . . . . 8. ¬(∃t:{t:T| ¬t = f(n-1) }. R(t))
| . . . . 9. ¬R(f(n-1))
| . . . . 10. t : T
| . . . . 11. R(t)
| . . . . ⊢   by Witness7 t
| . . . . |
| . . . . 7. ¬(∃t:{t:T| ¬t = f(n-1) }. R(t))
| . . . . 8. ¬R(f(n-1))
| . . . . 9. t : T
| . . . . 10. R(t)
| . . . . 11. ∃a:ℕn. f(a) = t
| . . . . ⊢   by Analyze 11
| . . . . |
| . . . . 11. a : ℕn
| . . . . 12. f(a) = t
| . . . . ⊢   by Decide (a = n-1)
| . . . . |
| . . . . |\
| . . . . | 13. a = n-1
| . . . . | ⊢   by RWH (RevHypC 13) 8
| . . . . | |
| . . . . | 8. t : T
| . . . . | 9. R(t)
| . . . . | 10. a : ℕn
| . . . . | 11. f(a) = t
| . . . . | 12. a = n-1
| . . . . | 13. ¬R(f(a))
| . . . . | ⊢   by ApFunToHypEquands ‘x’ (R(x)) Prop 11
| . . . . | |
| . . . . | 14. R(f(a)) = R(t)
| . . . . | ⊢   by RWH (HypC 14) 13
| . . . . |
| . . . . .\
| . . . . . 13. ¬a = n-1
| . . . . . ⊢   by Analyze 7
| . . . . . |
| . . . . . 7. ¬R(f(n-1))
| . . . . . 8. t : T
| . . . . . 9. R(t)
| . . . . . 10. a : ℕn
| . . . . . 11. f(a) = t
| . . . . . 12. ¬a = n-1
| . . . . . ⊢  ∃t:{t:T| ¬t = f(n-1) }. R(t)  by Witness t
| . . . . . |
| . . . . . .....wf .....
| . . . . . ⊢  t ∈ {t:T| ¬t = f(n-1) }  by Analyze THEN Try Auto
| . . . . . |
| . . . . . .....set predicate .....
| . . . . . ⊢  ¬t = f(n-1)  by Analyze 0
| . . . . . |
| . . . . . 13. t = f(n-1)
| . . . . . ⊢  False  by RWH (HypC 13) 11
| . . . . . |
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| . . . . . 11. f(a) = f(n-1)
| . . . . . 12. ¬a = n-1
| . . . . . 13. t = f(n-1)
| . . . . . ⊢   by Unfold ‘inject‘ 6
| . . . . . |
| . . . . . 6. ∀a1,a2:ℕn. f(a1) = f(a2) ⇒ a1 = a2
| . . . . . 7. ¬R(f(n-1))

| . . . . . 8. t : T
| . . . . . 9. R(t)
| . . . . . 10. a : ℕn
| . . . . . 11. f(a) = f(n-1)
| . . . . . 12. ¬a = n-1
| . . . . . 13. t = f(n-1)
| . . . . . ⊢   by InstHyp [a;n-1] 6 THEN Try Auto
|
.\
. 5. ∀n:ℕ, T:Type, R:(T→Prop).
. |  (∃f:(ℕn→T). Bij(ℕn; T; f)) & (∀t:T. Dec(R(t))) ⇒
. |  Dec(∃t:T. R(t))
. ⊢   by Analyze 3
. |
. 3. n : ℕ
. 4. ∃f:(ℕn→T). Bij(ℕn; T; f)
. 5. ∀t:T. Dec(R(t))
. 6. ∀n:ℕ, T:Type, R:(T→Prop).
. |  (∃f:(ℕn→T). Bij(ℕn; T; f)) & (∀t:T. Dec(R(t))) ⇒
. |  Dec(∃t:T. R(t))
. ⊢   by Witness6 n
. |
. 6. ∀T:Type, R:(T→Prop).
. |  (∃f:(ℕn→T). Bij(ℕn; T; f)) & (∀t:T. Dec(R(t))) ⇒
. |  Dec(∃t:T. R(t))
. ⊢   by Witness6 T
. |
. 6. ∀R:(T→Prop).
. |  (∃f:(ℕn→T). Bij(ℕn; T; f)) & (∀t:T. Dec(R(t))) ⇒
. |  Dec(∃t:T. R(t))
. ⊢   by Witness6 R
. |
. 6. (∃f:(ℕn→T). Bij(ℕn; T; f)) & (∀t:T. Dec(R(t))) ⇒
. |  Dec(∃t:T. R(t))
. ⊢   by Analyze 6
. |
. |\
. | .....antecedent .....
. | 5. ∀t:T. Dec(R(t))
. | ⊢  (∃f:(ℕn→T). Bij(ℕn; T; f)) & (∀t:T. Dec(R(t)))  by Analyze 0
. |
. .\
. . 6. Dec(∃t:T. R(t))
. . ⊢   by Auto
END
```