```
auto2_lemma_8  THM

⊢  ∀R:(Alph*→Alph*→Prop), n:ℕ+, L:(Alph*→𝔹), m:ℕ.
|  (∀x:Alph*. R(x,x))
|  & (∀x,y:Alph*. R(x,y) ⇒ R(y,x))
|  & (∀x,y,z:Alph*. R(x,y) & R(y,z) ⇒ R(x,z))
|  & (∀x,y,z:Alph*. R(x,y) ⇒ R((z @ x),z @ y))
|  & (∃w:(ℕn→Alph*). ∀l:Alph*. ∃i:ℕn. R(l,w(i)))
|  & (∃v:(ℕm→Alph*). ∀l:Alph*. L(l) ⟺ (∃i:ℕm. R(l,v(i))))
|  & Fin(Alph)
|  ⇒
|  (∀x,y:Alph*. Dec(∀l:Alph*. L(l @ x) = L(l @ y)))  by UnivCD
|
1. Alph : Type
2. R : Alph*→Alph*→Prop
3. n : ℕ+
4. L : Alph*→𝔹
5. m : ℕ
6. (∀x:Alph*. R(x,x))
|  & (∀x,y:Alph*. R(x,y) ⇒ R(y,x))
|  & (∀x,y,z:Alph*. R(x,y) & R(y,z) ⇒ R(x,z))
|  & (∀x,y,z:Alph*. R(x,y) ⇒ R((z @ x),z @ y))
|  & (∃w:(ℕn→Alph*). ∀l:Alph*. ∃i:ℕn. R(l,w(i)))
|  & (∃v:(ℕm→Alph*). ∀l:Alph*. L(l) ⟺ (∃i:ℕm. R(l,v(i))))
|  & Fin(Alph)
7. x : Alph*
8. y : Alph*
⊢  Dec(∀l:Alph*. L(l @ x) = L(l @ y))
| by Inst
|    Thm* ∀P:(T→Prop).
|         (∀x:T. Dec(P(x))) & Dec(∃x:T. ¬P(x)) ⇒ Dec(∀x:T. P(x))
|    {auto2_lemma_0}
|    [Alph*;λl.L(l @ x) = L(l @ y)]
|
|\
| .....decidable? .....
| 6. ∀x:Alph*. R(x,x)
| 7. ∀x,y:Alph*. R(x,y) ⇒ R(y,x)
| 8. ∀x,y,z:Alph*. R(x,y) & R(y,z) ⇒ R(x,z)
| 9. ∀x,y,z:Alph*. R(x,y) ⇒ R((z @ x),z @ y)
| 10. ∃w:(ℕn→Alph*). ∀l:Alph*. ∃i:ℕn. R(l,w(i))
| 11. ∃v:(ℕm→Alph*). ∀l:Alph*. L(l) ⟺ (∃i:ℕm. R(l,v(i)))
| 12. Fin(Alph)
| 13. x : Alph*
| 14. y : Alph*
| 15. x@0 : Alph*
| ⊢  Dec((λl.L(l @ x) = L(l @ y))(x@0))  by Reduce 0
| |
| ⊢  Dec(L(x@0 @ x) = L(x@0 @ y))  by ProveDecidable
|
|\
| .....decidable? .....
| 6. ∀x:Alph*. R(x,x)
| 7. ∀x,y:Alph*. R(x,y) ⇒ R(y,x)
| 8. ∀x,y,z:Alph*. R(x,y) & R(y,z) ⇒ R(x,z)
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| 9. ∀x,y,z:Alph*. R(x,y) ⇒ R((z @ x),z @ y)
| 10. ∃w:(ℕn→Alph*). ∀l:Alph*. ∃i:ℕn. R(l,w(i))
| 11. ∃v:(ℕm→Alph*). ∀l:Alph*. L(l) ⟺ (∃i:ℕm. R(l,v(i)))
| 12. Fin(Alph)
| 13. x : Alph*
| 14. y : Alph*
| ⊢  Dec(∃x@0:Alph*. ¬(λl.L(l @ x) = L(l @ y))(x@0))  by Reduce 0
| |
| ⊢  Dec(∃x@0:Alph*. ¬L(x@0 @ x) = L(x@0 @ y))
| | by Inst
| |    Thm* ∀R:(Alph*→Alph*→Prop), n:ℕ+, L:(Alph*→𝔹), m:ℕ.
| |          (∀x:Alph*. R(x,x))
| |          & (∀x,y:Alph*. R(x,y) ⇒ R(y,x))
| |          & (∀x,y,z:Alph*. R(x,y) & R(y,z) ⇒ R(x,z))
| |          & (∀x,y,z:Alph*. R(x,y) ⇒ R((z @ x),z @ y))
| |          & (∃w:(ℕn→Alph*). ∀l:Alph*. ∃i:ℕn. R(l,w(i)))
| |          & (∃v:(ℕm→Alph*). ∀l:Alph*. L(l) ⟺ (∃i:ℕm. R(l,v(i))))
| |          & Fin(Alph)
| |          ⇒
| |          (∀x,y:Alph*. Dec(∃l:Alph*. ¬L(l @ x) = L(l @ y)))
| |    {auto2_lemma_7}
| |    [Alph;R;n;L;m]
| |
| |\
| | .....antecedent .....
| | ⊢  (∀x:Alph*. R(x,x))
| | |  & (∀x,y:Alph*. R(x,y) ⇒ R(y,x))
| | |  & (∀x,y,z:Alph*. R(x,y) & R(y,z) ⇒ R(x,z))
| | |  & (∀x,y,z:Alph*. R(x,y) ⇒ R((z @ x),z @ y))
| | |  & (∃w:(ℕn→Alph*). ∀l:Alph*. ∃i:ℕn. R(l,w(i)))
| | |  & (∃v:(ℕm→Alph*). ∀l:Alph*. L(l) ⟺ (∃i:ℕm. R(l,v(i))))
| | |  & Fin(Alph)  by UnivCD
| | |
| | ⊢   by Analyze 0
| | |
| | |\
| | | ⊢  ∀x:Alph*. R(x,x)  by Auto
| | |
| | .\
| | . ⊢  (∀x,y:Alph*. R(x,y) ⇒ R(y,x))
| | . |  & (∀x,y,z:Alph*. R(x,y) & R(y,z) ⇒ R(x,z))
| | . |  & (∀x,y,z:Alph*. R(x,y) ⇒ R((z @ x),z @ y))
| | . |  & (∃w:(ℕn→Alph*). ∀l:Alph*. ∃i:ℕn. R(l,w(i)))
| | . |  & (∃v:(ℕm→Alph*). ∀l:Alph*. L(l) ⟺ (∃i:ℕm. R(l,v(i))))
| | . |  & Fin(Alph)  by Analyze 0
| | . |
| | . |\
| | . | ⊢  ∀x,y:Alph*. R(x,y) ⇒ R(y,x)  by Auto
| | . |
| | . .\
| | . . ⊢  (∀x,y,z:Alph*. R(x,y) & R(y,z) ⇒ R(x,z))
| | . . |  & (∀x,y,z:Alph*. R(x,y) ⇒ R((z @ x),z @ y))
| | . . |  & (∃w:(ℕn→Alph*). ∀l:Alph*. ∃i:ℕn. R(l,w(i)))
| | . . |  & (∃v:(ℕm→Alph*). ∀l:Alph*. L(l) ⟺ (∃i:ℕm. R(l,v(i))))
| | . . |  & Fin(Alph)  by Analyze 0
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| | . . |
| | . . |\
| | . . | ⊢  ∀x,y,z:Alph*. R(x,y) & R(y,z) ⇒ R(x,z)  by Auto
| | . . |
| | . . .\
| | . . . ⊢  (∀x,y,z:Alph*. R(x,y) ⇒ R((z @ x),z @ y))
| | . . . |  & (∃w:(ℕn→Alph*). ∀l:Alph*. ∃i:ℕn. R(l,w(i)))
| | . . . |  & (∃v:(ℕm→Alph*). ∀l:Alph*. L(l) ⟺ (∃i:ℕm. R(l,v(i))))
| | . . . |  & Fin(Alph)  by Analyze 0
| | . . . |
| | . . . |\
| | . . . | ⊢  ∀x,y,z:Alph*. R(x,y) ⇒ R((z @ x),z @ y)  by Auto
| | . . . |
| | . . . .\
| | . . . . ⊢  (∃w:(ℕn→Alph*). ∀l:Alph*. ∃i:ℕn. R(l,w(i)))
| | . . . . |  & (∃v:(ℕm→Alph*).
| | . . . . |     ∀l:Alph*. L(l) ⟺ (∃i:ℕm. R(l,v(i))))
| | . . . . |  & Fin(Alph)  by Analyze 0
| | . . . . |
| | . . . . |\
| | . . . . | ⊢  ∃w:(ℕn→Alph*). ∀l:Alph*. ∃i:ℕn. R(l,w(i))  by Auto
| | . . . . |
| | . . . . .\
| | . . . . . ⊢  (∃v:(ℕm→Alph*).
| | . . . . . |   ∀l:Alph*. L(l) ⟺ (∃i:ℕm. R(l,v(i))))
| | . . . . . |  & Fin(Alph)  by Analyze 0
| | . . . . . |
| | . . . . . |\
| | . . . . . | ⊢  ∃v:(ℕm→Alph*).
| | . . . . . |     ∀l:Alph*. L(l) ⟺ (∃i:ℕm. R(l,v(i)))  by Auto
| | . . . . . |
| | . . . . . .\
| | . . . . . . ⊢  Fin(Alph)  by Auto
| |
| .\
| . 15. ∀x,y:Alph*. Dec(∃l:Alph*. ¬L(l @ x) = L(l @ y))
| . ⊢   by InstHyp [x;y] 15
|
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. 9. Dec(∀x@0:Alph*. (λl.L(l @ x) = L(l @ y))(x@0))
. ⊢   by Reduce 9
END
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