```
en_inj  THM

⊢  ∀n:ℕ+, m:ℕ, l1,l2:ℕn*.
|  ||l1|| = m & ||l2|| = m ⇒ en(l1) = en(l2) ⇒ l1 = l2
| by Analyze 0 THEN Analyze 0
|
1. n : ℕ+
2. m : ℕ
⊢  ∀l1,l2:ℕn*. ||l1|| = m & ||l2|| = m ⇒ en(l1) = en(l2) ⇒ l1 = l2
| by NatInd 2
|
|\
| .....basecase .....
| 1. n : ℕ+
| ⊢  ∀l1,l2:ℕn*. ||l1|| = 0 & ||l2|| = 0 ⇒ en(l1) = en(l2) ⇒ l1 = l2
| | by UnivCD
| |
| 2. l1 : ℕn*
| 3. l2 : ℕn*
| 4. ||l1|| = 0 & ||l2|| = 0
| 5. en(l1) = en(l2)
| ⊢  l1 = l2
|   by RWH
|      (LemmaC
|       Thm* ∀l:T*. ||l|| = 0 ⇒ l = nil    {zero_length_imp_nil})
|      0
|
.\
. .....upcase .....
. 2. m : ℤ
. 3. 0<m
. 4. ∀l1,l2:ℕn*.
. |  ||l1|| = m-1 & ||l2|| = m-1 ⇒ en(l1) = en(l2) ⇒ l1 = l2
. ⊢   by Analyze 0 THEN Analyze 0
. |
. 5. l1 : ℕn*
. 6. l2 : ℕn*
. ⊢  ||l1|| = m & ||l2|| = m ⇒ en(l1) = en(l2) ⇒ l1 = l2
. | by Analyze 5
. |
. |\
. | ⊢  ||nil|| = m & ||l2|| = m ⇒ en(nil) = en(l2) ⇒ nil = l2
. | | by Analyze 6
. | |
. | |\
. | | ⊢  ||nil|| = m & ||nil|| = m ⇒ en(nil) = en(nil) ⇒ nil = nil
. | |   by Auto
. | |
. | .\
. | . 7. u : ℕn
. | . 8. v : ℕn*
. | . ⊢  ||nil|| = m & ||u.v|| = m ⇒ en(nil) = en(u.v) ⇒ nil = u.v
. | .   by Reduce 0
. |
. .\
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. . 7. u : ℕn
. . 8. v : ℕn*
. . ⊢  ||u.v|| = m & ||l2|| = m ⇒ en(u.v) = en(l2) ⇒ u.v = l2
. . | by Analyze 6
. . |
. . |\
. . | ⊢  ||u.v|| = m & ||nil|| = m ⇒ en(u.v) = en(nil) ⇒ u.v = nil
. . |   by Reduce 0
. . |
. . .\
. . . 9. u1 : ℕn
. . . 10. v1 : ℕn*
. . . ⊢  ||u.v|| = m & ||u1.v1|| = m ⇒
. . . |  en(u.v) = en(u1.v1) ⇒ u.v = u1.v1  by Thin 6 THEN Thin 5
. . . |
. . . 5. u : ℕn
. . . 6. v : ℕn*
. . . 7. u1 : ℕn
. . . 8. v1 : ℕn*
. . . ⊢   by UnivCD
. . . |
. . . 9. ||u.v|| = m & ||u1.v1|| = m
. . . 10. en(u.v) = en(u1.v1)
. . . ⊢  u.v = u1.v1  by RecUnfold ‘en‘ 10 THEN Reduce 10
. . . |
. . . 10. u+en(v)·n = u1+en(v1)·n
. . . ⊢
. . . | by Inst
. . . |    Thm* ∀n:ℕ+, r1,r2:ℕn, q1,q2:ℕ.
. . . |         r1+q1·n = r2+q2·n ⇒ r1 = r2 & q1 = q2
. . . |    {rem_quo_unique}
. . . |    [n;u;u1;en(v);en(v1)]
. . . |
. . . 11. u = u1 & en(v) = en(v1)
. . . ⊢   by Analyze
. . . |
. . . |\
. . . | ⊢  u = u1  by Auto
. . . |
. . . .\
. . . . ⊢  v = v1  by BackThru 4
END
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