```
en_surj  THM

⊢  ∀n:ℕ+, m:ℕ, k:ℕ(n↑m). ∃l:ℕn*. ||l|| = m & en(l) = k
| by Analyze 0 THEN Analyze 0 THEN NatInd 2
|
|\
| .....basecase .....
| 1. n : ℕ+
| ⊢  ∀k:ℕ(n↑0). ∃l:ℕn*. ||l|| = 0 & en(l) = k  by Analyze 0
| |
| 2. k : ℕ(n↑0)
| ⊢  ∃l:ℕn*. ||l|| = 0 & en(l) = k
| | by RecUnfold ‘exp‘ 2 THEN Reduce 2
| |
| 2. k : ℕ1
| ⊢   by Witness nil
| |
| ⊢  ||nil|| = 0 & en(nil) = k  by RecUnfold ‘en‘ 0 THEN Reduce 0
|
.\
. .....upcase .....
. 1. n : ℕ+
. 2. m : ℤ
. 3. 0<m
. 4. ∀k:ℕ(n↑m-1). ∃l:ℕn*. ||l|| = m-1 & en(l) = k
. ⊢  ∀k:ℕ(n↑m). ∃l:ℕn*. ||l|| = m & en(l) = k  by Analyze 0
. |
. 5. k : ℕ(n↑m)
. ⊢  ∃l:ℕn*. ||l|| = m & en(l) = k  by Witness4 k ÷ n
. |
. |\
. | 4. k : ℕ(n↑m)
. | ⊢  0≤(k ÷ n)
. |   by BackThru Thm* ∀a:ℕ, n:ℕ+. 0≤(a ÷ n)    {div_bounds_1}
. |
. |\
. | 4. k : ℕ(n↑m)
. | ⊢  (k ÷ n)<(n↑m-1)  by Analyze 4
. | |
. | 4. k : ℤ
. | 5. 0 ≤ k < (n↑m)
. | ⊢   by Analyze 5
. | |
. | 5. 0≤k
. | 6. k<(n↑m)
. | ⊢
. | | by RWH
. | |    (LemmaC Thm* ∀n,k:ℕ. 0<k ⇒ (n↑k) = n·(n↑k-1)    {exp_step})
. | |    6
. | |
. | 6. k<n·(n↑m-1)
. | ⊢
. |   by BackThru
. |      Thm* ∀a:ℕ, n:ℕ+, k:ℕ. a<n·k ⇒ (a ÷ n)<k    {div_lt_lbound}
. |
```

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. .\
. . 4. k : ℕ(n↑m)
. . 5. ∃l:ℕn*. ||l|| = m-1 & en(l) = (k ÷ n)
. . ⊢   by Analyze 5
. . |
. . 5. l : ℕn*
. . 6. ||l|| = m-1 & en(l) = (k ÷ n)
. . ⊢
. . | by Inst
. . |    Thm* ∀a:ℕ, n:ℕ+. 0≤(a rem n) & (a rem n)<n    {rem_bounds_1}
. . |    [k;n]
. . |    THEN
. . |    Witness (k rem n).l
. . |
. . 7. 0≤(k rem n) & (k rem n)<n
. . ⊢  ||(k rem n).l|| = m & en((k rem n).l) = k
. . | by RecUnfold ‘en‘ 0 THEN Reduce 0
. . |
. . 6. ||l|| = m-1
. . 7. en(l) = (k ÷ n)
. . 8. 0≤(k rem n)
. . 9. (k rem n)<n
. . ⊢  (k rem n)+en(l)·n = k  by RWH (HypC 7) 0
. . |
. . ⊢  (k rem n)+(k ÷ n)·n = k
. . | by RWH (LemmaC Thm* ∀a,b:ℤ. a+b = b+a    {add_com}) 0
. . |
. . ⊢  (k ÷ n)·n+(k rem n) = k
. .   by RWH
. .      (RevLemmaC
. .       Thm* ∀a:ℤ, n:ℤ-0. a = (a ÷ n)·n+(a rem n)    {div_rem_sum})
. .      0
END
```