```
enum_surj  THM

⊢  ∀q:ℕ^+, a:ℕ. ∃l:ℕq*. enum(l) = a  by UnivCD
|
1. q : ℕ^+
2. a : ℕ
⊢  ∃l:ℕq*. enum(l) = a
| by Inst
|     Thm* ∀q:ℕ^+, a:ℕ. ∃n:ℕ. Σ(q↑n^-) ≤ a < Σ(q↑n+1^-)
|     {geom_series_limit}
|     [q;a]
|     THEN
|     Analyze 3
|
3. n : ℕ
4. Σ(q↑n^-) ≤ a < Σ(q↑n+1^-)
⊢
| by Inst
|     Thm* ∀n:ℕ^+, m:ℕ, k:ℕ(n↑m). ∃l:ℕn*. ||l|| = m & en(l) = k
|     {en_surj}
|     [q;n;a-Σ(q↑n^-)]
|
|\
| 4. Σ(q↑n^-)≤a
| 5. a<Σ(q↑n+1^-)
| ⊢  0≤a-Σ(q↑n^-)  by Auto
|
|\
| 4. Σ(q↑n^-)≤a
| 5. a<Σ(q↑n+1^-)
| ⊢  a-Σ(q↑n^-)<(q↑n)
| | by RWH
| |     (LemmaC
| |      Thm* ∀q,n:ℕ. Σ(q↑n+1^-) = Σ(q↑n^-)+(q↑n) ∈ ℕ
| |      {geom_series_step})
| |     5
| |
| 5. a<Σ(q↑n^-)+(q↑n)
| ⊢   by Auto
|
.\
. 5. ∃l:ℕq*. ||l|| = n & en(l) = a-Σ(q↑n^-)
. ⊢   by Analyze 5
. |
. 5. l : ℕq*
. 6. ||l|| = n & en(l) = a-Σ(q↑n^-)
. ⊢   by Witness l
. |
. ⊢  enum(l) = a  by Unfold ‘enum‘ 0
. |
. ⊢  Σ(q↑||l||^-)+en(l) = a  by Analyze 6
. |
. 6. ||l|| = n
. 7. en(l) = a-Σ(q↑n^-)
. ⊢   by Assert (||l|| = n)
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. 8. ||l|| = n
. ⊢   by RWH (HypC 8) 0
END
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