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fun_enumer  THM

⊢  ∀n,m:ℕ. ∃f:((ℕm→ℕn)→ℕ(n↑m)). Bij(ℕm→ℕn; ℕ(n↑m); f)  by UnivCD
|
1. n : ℕ
2. m : ℕ
⊢  ∃f:((ℕm→ℕn)→ℕ(n↑m)). Bij(ℕm→ℕn; ℕ(n↑m); f)  by NatInd 2
|
|\
| .....basecase .....
| 1. n : ℕ
| ⊢  ∃f:((ℕ0→ℕn)→ℕ(n↑0)). Bij(ℕ0→ℕn; ℕ(n↑0); f)  by Witness λz.0
| |
| |\
| | 2. ℕ0→ℕn
| | ⊢  0<(n↑0)  by RecCaseSplit ‘exp‘
| |
| .\
| . ⊢  Bij(ℕ0→ℕn; ℕ(n↑0); λz.0)  by Unfold ‘biject‘ 0
| . |
| . ⊢  Inj(ℕ0→ℕn; ℕ(n↑0); λz.0) & Surj(ℕ0→ℕn; ℕ(n↑0); λz.0)
| . | by Analyze 0
| . |
| . |\
| . | ⊢  Inj(ℕ0→ℕn; ℕ(n↑0); λz.0)  by Unfold ‘inject‘ 0
| . | |
| . | ⊢  ∀a1,a2:(ℕ0→ℕn). (λz.0)(a1) = (λz.0)(a2) ⇒ a1 = a2
| . | | by Analyze 0 THEN Analyze 0
| . | |
| . | 2. a1 : ℕ0→ℕn
| . | 3. a2 : ℕ0→ℕn
| . | ⊢  (λz.0)(a1) = (λz.0)(a2) ⇒ a1 = a2  by Reduce 0
| . | |
| . | ⊢  0 = 0 ⇒ a1 = a2  by Analyze 0
| . | |
| . | |\
| . | | 4. 0 = 0
| . | | ⊢  a1 = a2  by Ext
| . | | |
| . | | 5. x : ℕ0
| . | | ⊢  a1(x) = a2(x)  by Analyze 5
| . | |
| . | .\
| . | . .....wf .....
| . | . ⊢  0 = 0 ∈ Prop  by Auto
| . | . |
| . | . |\
| . | . | ⊢  0<(n↑0)  by RecCaseSplit ‘exp‘
| . | . |
| . | . .\
| . | . . ⊢  0<(n↑0)  by RecCaseSplit ‘exp‘
| . |
| . .\
| . . ⊢  Surj(ℕ0→ℕn; ℕ(n↑0); λz.0)  by Unfold ‘surject‘ 0
| . . |
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| . . ⊢  ∀b:ℕ(n↑0). ∃a:(ℕ0→ℕn). (λz.0)(a) = b  by Analyze 0
| . . |
| . . 2. b : ℕ(n↑0)
| . . ⊢  ∃a:(ℕ0→ℕn). (λz.0)(a) = b  by Reduce 0
| . . |
| . . ⊢  ∃a:(ℕ0→ℕn). 0 = b  by Witness λz.0
| . . |
| . . ⊢  0 = b  by Analyze 2
| . . |
| . . 2. b : ℤ
| . . 3. 0 ≤ b < (n↑0)
| . . ⊢   by RWH (RecUnfoldC ‘exp‘) 3
| . . |
| . . 3. 0 ≤ b < if 0=_2 0→ 1 else n·(n↑0-1) fi
| . . ⊢   by Reduce 3
| . . |
| . . 3. 0 ≤ b < 1
| . . ⊢   by Auto
| . . |
| . . 1. n : ℤ
| . . 2. 0≤n
| . . 3. b : ℤ
| . . 4. 0≤b
| . . 5. b<1
| . . ⊢  0<(n↑0)  by RecCaseSplit ‘exp‘
|
.\
. .....upcase .....
. 2. m : ℤ
. 3. 0<m
. 4. ∃f:((ℕ(m-1)→ℕn)→ℕ(n↑m-1)). Bij(ℕ(m-1)→ℕn; ℕ(n↑m-1); f)
. ⊢   by Analyze 4
. |
. 4. f : (ℕ(m-1)→ℕn)→ℕ(n↑m-1)
. 5. Bij(ℕ(m-1)→ℕn; ℕ(n↑m-1); f)
. ⊢   by Decide (n = 0)
. |
. |\
. | 6. n = 0
. | ⊢   by Witness λz.z(0)
. | |
. | ⊢  Bij(ℕm→ℕn; ℕ(n↑m); λz.z(0))
. | | by Unfold ‘biject‘ 0 THEN Analyze 0
. | |
. | |\
. | | ⊢  Inj(ℕm→ℕn; ℕ(n↑m); λz.z(0))  by Unfold ‘inject‘ 0
. | | |
. | | ⊢  ∀a1,a2:(ℕm→ℕn). (λz.z(0))(a1) = (λz.z(0))(a2) ⇒ a1 = a2
. | | | by Reduce 0
. | | |
. | | ⊢  ∀a1,a2:(ℕm→ℕn). a1(0) = a2(0) ⇒ a1 = a2  by UnivCD
. | | |
. | | 7. a1 : ℕm→ℕn
. | | 8. a2 : ℕm→ℕn
. | | 9. a1(0) = a2(0)
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. | | ⊢  a1 = a2  by Ext
. | | |
. | | 10. x : ℕm
. | | ⊢  a1(x) = a2(x)  by Analyze 9
. | | |
. | | 9. a1(0) = a2(0)
. | | 10. 0 ≤ a1(0) < (n↑m)
. | | 11. x : ℕm
. | | ⊢   by Inst Thm* ∀m:ℕ+. (0↑m) = 0    {exp_zero} [m]
. | |
. | .\
. | . ⊢  Surj(ℕm→ℕn; ℕ(n↑m); λz.z(0))  by Unfold ‘surject‘ 0
. | . |
. | . ⊢  ∀b:ℕ(n↑m). ∃a:(ℕm→ℕn). (λz.z(0))(a) = b  by Analyze 0
. | . |
. | . 7. b : ℕ(n↑m)
. | . ⊢  ∃a:(ℕm→ℕn). (λz.z(0))(a) = b  by Analyze 7
. | . |
. | . 7. b : ℤ
. | . 8. 0 ≤ b < (n↑m)
. | . ⊢   by Assert ((n↑m) = 0)
. | . |
. | . |\
. | . | .....assertion .....
. | . | ⊢  (n↑m) = 0  by RWH (HypC 6) 0
. | . | |
. | . | ⊢  (0↑m) = 0
. | . |   by RWH (LemmaC Thm* ∀m:ℕ+. (0↑m) = 0    {exp_zero}) 0
. | . |
. | . .\
. | . . 9. (n↑m) = 0
. | . . ⊢   by Auto
. |
. .\
. . 6. ¬n = 0
. . ⊢
. . | by Inst
. . |    Thm* ∀n,m:ℕ+. ∃f:(ℕn×ℕm→ℕ(n·m)). Bij(ℕn×ℕm; ℕ(n·m); f)
. . |    {rect_enumer}
. . |    [n;(n↑m-1)]
. . |
. . |\
. . | .....set predicate .....
. . | ⊢  0<(n↑m-1)
. . |   by Inst Thm* ∀m:ℕ+, n:ℕ. 0<(m↑n)    {exp_non_zero} [n;m-1]
. . |
. . .\
. . . 7. ∃f:(ℕn×ℕ(n↑m-1)→ℕ(n·(n↑m-1))).
. . . |  Bij(ℕn×ℕ(n↑m-1); ℕ(n·(n↑m-1)); f)
. . . ⊢   by Analyze 7
. . . |
. . . 7. f1 : ℕn×ℕ(n↑m-1)→ℕ(n·(n↑m-1))
. . . 8. Bij(ℕn×ℕ(n↑m-1); ℕ(n·(n↑m-1)); f1)
. . . ⊢   by Assert (n·(n↑m-1) = (n↑m))
. . . |
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. . . |\
. . . | .....assertion .....
. . . | ⊢  n·(n↑m-1) = (n↑m)
. . . | | by RW (AddrC [3] (RecUnfoldTopC ‘exp‘)) 0
. . . | |
. . . | ⊢  n·(n↑m-1) = if m=_2 0→ 1 else n·(n↑m-1) fi
. . . |   by SplitOnConclITE
. . . |
. . . .\
. . . . 9. n·(n↑m-1) = (n↑m)
. . . . ⊢   by HypSubst 9 7 THEN Thin 9
. . . . |
. . . . 7. f1 : ℕn×ℕ(n↑m-1)→ℕ(n↑m)
. . . . 8. Bij(ℕn×ℕ(n↑m-1); ℕ(n·(n↑m-1)); f1)
. . . . ⊢   by Witness λz.f1(<z(m-1),f(z)>)
. . . . |
. . . . ⊢  Bij(ℕm→ℕn; ℕ(n↑m); λz.f1(<z(m-1),f(z)>))
. . . . | by Unfold ‘biject‘ 0 THEN Analyze 0
. . . . |
. . . . |\
. . . . | ⊢  Inj(ℕm→ℕn; ℕ(n↑m); λz.f1(<z(m-1),f(z)>))
. . . . | | by Unfold ‘inject‘ 0 THEN Reduce 0 THEN UnivCD
. . . . | |
. . . . | 9. a1 : ℕm→ℕn
. . . . | 10. a2 : ℕm→ℕn
. . . . | 11. f1(<a1(m-1),f(a1)>) = f1(<a2(m-1),f(a2)>)
. . . . | ⊢  a1 = a2  by Unfold ‘biject‘ 8 THEN Analyze 8 THEN Thin 9
. . . . | |
. . . . | 8. Inj(ℕn×ℕ(n↑m-1); ℕ(n·(n↑m-1)); f1)
. . . . | 9. a1 : ℕm→ℕn
. . . . | 10. a2 : ℕm→ℕn
. . . . | 11. f1(<a1(m-1),f(a1)>) = f1(<a2(m-1),f(a2)>)
. . . . | ⊢   by Unfold ‘inject‘ 8
. . . . | |
. . . . | 8. ∀a1,a2:(ℕn×ℕ(n↑m-1)). f1(a1) = f1(a2) ⇒ a1 = a2
. . . . | 9. a1 : ℕm→ℕn
. . . . | 10. a2 : ℕm→ℕn
. . . . | 11. f1(<a1(m-1),f(a1)>) = f1(<a2(m-1),f(a2)>)
. . . . | ⊢   by InstHyp [<a1(m-1),f(a1)>;<a2(m-1),f(a2)>] 8
. . . . | |
. . . . | |\
. . . . | | .....antecedent .....
. . . . | | ⊢  f1(<a1(m-1),f(a1)>) = f1(<a2(m-1),f(a2)>)
. . . . | |   by RWH
. . . . | |      (LemmaC
. . . . | |       Thm* ∀n,k:ℕ. 0<k ⇒ (n↑k) = n·(n↑k-1)    {exp_step})
. . . . | |      11
. . . . | |
. . . . | .\
. . . . | . 12. <a1(m-1),f(a1)> = <a2(m-1),f(a2)>
. . . . | . ⊢   by Analyze 12 THEN Reduce 12 THEN Reduce 13
. . . . | . |
. . . . | . 12. a1(m-1) = a2(m-1)
. . . . | . 13. f(a1) = f(a2)
. . . . | . ⊢   by Ext
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. . . . | . |
. . . . | . 14. x : ℕm
. . . . | . ⊢  a1(x) = a2(x)  by Decide (x = m-1)
. . . . | . |
. . . . | . |\
. . . . | . | 15. x = m-1
. . . . | . | ⊢   by RWH (HypC 15) 0
. . . . | . |
. . . . | . .\
. . . . | . . 15. ¬x = m-1
. . . . | . . ⊢   by Unfold ‘biject‘ 5 THEN Analyze 5 THEN Thin 6
. . . . | . . |
. . . . | . . 5. Inj(ℕ(m-1)→ℕn; ℕ(n↑m-1); f)
. . . . | . . 6. ¬n = 0
. . . . | . . 7. f1 : ℕn×ℕ(n↑m-1)→ℕ(n↑m)
. . . . | . . 8. ∀a1,a2:(ℕn×ℕ(n↑m-1)). f1(a1) = f1(a2) ⇒ a1 = a2
. . . . | . . 9. a1 : ℕm→ℕn
. . . . | . . 10. a2 : ℕm→ℕn
. . . . | . . 11. f1(<a1(m-1),f(a1)>) = f1(<a2(m-1),f(a2)>)
. . . . | . . 12. a1(m-1) = a2(m-1)
. . . . | . . 13. f(a1) = f(a2)
. . . . | . . 14. x : ℕm
. . . . | . . 15. ¬x = m-1
. . . . | . . ⊢   by Unfold ‘inject‘ 5
. . . . | . . |
. . . . | . . 5. ∀a1,a2:(ℕ(m-1)→ℕn). f(a1) = f(a2) ⇒ a1 = a2
. . . . | . . 6. ¬n = 0
. . . . | . . 7. f1 : ℕn×ℕ(n↑m-1)→ℕ(n↑m)
. . . . | . . 8. ∀a1,a2:(ℕn×ℕ(n↑m-1)). f1(a1) = f1(a2) ⇒ a1 = a2
. . . . | . . 9. a1 : ℕm→ℕn
. . . . | . . 10. a2 : ℕm→ℕn
. . . . | . . 11. f1(<a1(m-1),f(a1)>) = f1(<a2(m-1),f(a2)>)
. . . . | . . 12. a1(m-1) = a2(m-1)
. . . . | . . 13. f(a1) = f(a2)
. . . . | . . 14. x : ℕm
. . . . | . . 15. ¬x = m-1
. . . . | . . ⊢   by Witness5 a1
. . . . | . . |
. . . . | . . 5. ¬n = 0
. . . . | . . 6. f1 : ℕn×ℕ(n↑m-1)→ℕ(n↑m)
. . . . | . . 7. ∀a1,a2:(ℕn×ℕ(n↑m-1)). f1(a1) = f1(a2) ⇒ a1 = a2
. . . . | . . 8. a1 : ℕm→ℕn
. . . . | . . 9. a2 : ℕm→ℕn
. . . . | . . 10. f1(<a1(m-1),f(a1)>) = f1(<a2(m-1),f(a2)>)
. . . . | . . 11. a1(m-1) = a2(m-1)
. . . . | . . 12. f(a1) = f(a2)
. . . . | . . 13. x : ℕm
. . . . | . . 14. ¬x = m-1
. . . . | . . 15. ∀a2:(ℕ(m-1)→ℕn). f(a1) = f(a2) ⇒ a1 = a2
. . . . | . . ⊢   by Witness15 a2 THEN Analyze 15
. . . . | . . |
. . . . | . . 15. a1 = a2
. . . . | . . ⊢   by Assert (x ∈ ℕ(m-1))
. . . . | . . |
. . . . | . . |\
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. . . . | . . | .....assertion .....
. . . . | . . | ⊢  x ∈ ℕ(m-1)  by Auto
. . . . | . . | |
. . . . | . . | ⊢  x<m-1  by Auto
. . . . | . . |
. . . . | . . .\
. . . . | . . . 16. x ∈ ℕ(m-1)
. . . . | . . . ⊢   by HypSubst 15 0
. . . . | . . . |
. . . . | . . . |\
. . . . | . . . | ⊢  a2(x) = a2(x)  by Auto
. . . . | . . . |
. . . . | . . . .\
. . . . | . . . . .....wf .....
. . . . | . . . . 17. z : ℕ(m-1)→ℕn
. . . . | . . . . ⊢  z(x) = a2(x) ∈ Prop  by Auto
. . . . |
. . . . .\
. . . . . ⊢  Surj(ℕm→ℕn; ℕ(n↑m); λz.f1(<z(m-1),f(z)>))
. . . . . | by Unfold ‘surject‘ 0 THEN Analyze 0
. . . . . |
. . . . . 9. b : ℕ(n↑m)
. . . . . ⊢  ∃a:(ℕm→ℕn). (λz.f1(<z(m-1),f(z)>))(a) = b  by Reduce 0
. . . . . |
. . . . . ⊢  ∃a:(ℕm→ℕn). f1(<a(m-1),f(a)>) = b
. . . . . | by Unfold ‘biject‘ 8 THEN Analyze 8 THEN Thin 8
. . . . . |
. . . . . 8. Surj(ℕn×ℕ(n↑m-1); ℕ(n·(n↑m-1)); f1)
. . . . . 9. b : ℕ(n↑m)
. . . . . ⊢   by Unfold ‘surject‘ 8
. . . . . |
. . . . . 8. ∀b:ℕ(n·(n↑m-1)). ∃a:(ℕn×ℕ(n↑m-1)). f1(a) = b
. . . . . 9. b : ℕ(n↑m)
. . . . . ⊢   by Witness8 b
. . . . . |
. . . . . |\
. . . . . | .....wf .....
. . . . . | 8. b : ℕ(n↑m)
. . . . . | ⊢  b ∈ ℕ(n·(n↑m-1))
. . . . . | | by Inst
. . . . . | |    Thm* ∀n,k:ℕ. 0<k ⇒ (n↑k) = n·(n↑k-1)    {exp_step}
. . . . . | |    [n;m]
. . . . . | |
. . . . . | 9. (n↑m) = n·(n↑m-1)
. . . . . | ⊢   by RWH (RevHypC 9) 0
. . . . . |
. . . . . .\
. . . . . . 8. b : ℕ(n↑m)
. . . . . . 9. ∃a:(ℕn×ℕ(n↑m-1)). f1(a) = b
. . . . . . ⊢   by Analyze 9
. . . . . . |
. . . . . . 9. a : ℕn×ℕ(n↑m-1)
. . . . . . 10. f1(a) = b
. . . . . . ⊢   by Unfold ‘biject‘ 5 THEN Analyze 5 THEN Thin 5
. . . . . . |
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. . . . . . . 5. Surj(ℕ(m-1)→ℕn; ℕ(n↑m-1); f)
. . . . . . . 6. ¬n = 0
. . . . . . . 7. f1 : ℕn×ℕ(n↑m-1)→ℕ(n↑m)
. . . . . . . 8. b : ℕ(n↑m)
. . . . . . . 9. a : ℕn×ℕ(n↑m-1)
. . . . . . . 10. f1(a) = b
. . . . . . . ⊢   by Unfold `surject` 5
. . . . . . . |
. . . . . . . 5. ∀b:ℕ(n↑m-1). ∃a:(ℕ(m-1)→ℕn). f(a) = b
. . . . . . . 6. ¬n = 0
. . . . . . . 7. f1 : ℕn×ℕ(n↑m-1)→ℕ(n↑m)
. . . . . . . 8. b : ℕ(n↑m)
. . . . . . . 9. a : ℕn×ℕ(n↑m-1)
. . . . . . . 10. f1(a) = b
. . . . . . . ⊢   by Witness5 2of(a)
. . . . . . . |
. . . . . . . 5. ¬n = 0
. . . . . . . 6. f1 : ℕn×ℕ(n↑m-1)→ℕ(n↑m)
. . . . . . . 7. b : ℕ(n↑m)
. . . . . . . 8. a : ℕn×ℕ(n↑m-1)
. . . . . . . 9. f1(a) = b
. . . . . . . 10. ∃a@0:(ℕ(m-1)→ℕn). f(a@0) = 2of(a)
. . . . . . . ⊢   by Analyze 10
. . . . . . . |
. . . . . . . 10. a@0 : ℕ(m-1)→ℕn
. . . . . . . 11. f(a@0) = 2of(a)
. . . . . . . ⊢   by Witness λz.if z=_2m-1→ 1of(a) else a@0(z) fi
. . . . . . . |
. . . . . . . |\
. . . . . . . | .....wf .....
. . . . . . . | ⊢  (λz.if z=_2m-1→ 1of(a) else a@0(z) fi) ∈ ℕm→ℕn
. . . . . . . | | by Analyze
. . . . . . . | |
. . . . . . . | 12. z : ℕm
. . . . . . . | ⊢  if z=_2m-1→ 1of(a) else a@0(z) fi ∈ ℕn  by Analyze
. . . . . . . | |
. . . . . . . | |\
. . . . . . . | | ⊢  if z=_2m-1→ 1of(a) else a@0(z) fi ∈ ℤ
. . . . . . . | |   by SplitOnConclITE
. . . . . . . | |
. . . . . . . | .\
. . . . . . . | . .....set predicate .....
. . . . . . . | . ⊢  0 ≤ if z=_2m-1→ 1of(a) else a@0(z) fi < n
. . . . . . . | .   by SplitOnConclITE
. . . . . . . |
. . . . . . . |\
. . . . . . . | ⊢  f1
. . . . . . . | |  (<(λz.if z=_2m-1→ 1of(a) else a@0(z) fi)(m-1)
. . . . . . . | |   ,f(λz.if z=_2m-1→ 1of(a) else a@0(z) fi)>)
. . . . . . . | |  =
. . . . . . . | |  b  by Reduce 0
. . . . . . . | |
. . . . . . . | ⊢  f1
. . . . . . . | |  (<if m-1=_2m-1→ 1of(a) else a@0(m-1) fi
. . . . . . . | |   ,f(λz.if z=_2m-1→ 1of(a) else a@0(z) fi)>)
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. . . . . . . | |  =
. . . . . . . | |  b  by SplitOnConclITE
. . . . . . . | |
. . . . . . . | |\
. . . . . . . | | .....truecase .....
. . . . . . . | | 12. m-1 = m-1
. . . . . . . | | ⊢  f1(<1of(a),f(λz.if z=_2m-1→ 1of(a) else a@0(z) fi)>)
. . . . . . . | | |  =
. . . . . . . | | |  b
. . . . . . . | | | by Assert
. . . . . . . | | |    ((λz.if z=_2m-1→ 1of(a) else a@0(z) fi) = a@0)
. . . . . . . | | |
. . . . . . . | | |\
. . . . . . . | | | .....assertion .....
. . . . . . . | | | ⊢  (λz.if z=_2m-1→ 1of(a) else a@0(z) fi) = a@0
. . . . . . . | | | | by Ext
. . . . . . . | | | |
. . . . . . . | | | 13. x : ℕ(m-1)
. . . . . . . | | | ⊢  (λz.if z=_2m-1→ 1of(a) else a@0(z) fi)(x) = a@0(x)
. . . . . . . | | | | by Reduce 0
. . . . . . . | | | |
. . . . . . . | | | ⊢  if x=_2m-1→ 1of(a) else a@0(x) fi = a@0(x)
. . . . . . . | | | | by SplitOnConclITE
. . . . . . . | | | |
. . . . . . . | | | |\
. . . . . . . | | | | .....truecase .....
. . . . . . . | | | | 14. x = m-1
. . . . . . . | | | | ⊢  1of(a) = a@0(x)  by Auto
. . . . . . . | | | |
. . . . . . . | | | .\
. . . . . . . | | | . .....falsecase .....
. . . . . . . | | | . 14. ¬x = m-1
. . . . . . . | | | . ⊢  a@0(x) = a@0(x)  by Auto
. . . . . . . | | |
. . . . . . . | | .\
. . . . . . . | | . 13. (λz.if z=_2m-1→ 1of(a) else a@0(z) fi) = a@0
. . . . . . . | | . ⊢   by HypSubst 13 0
. . . . . . . | | . |
. . . . . . . | | . ⊢  f1(<1of(a),f(a@0)>) = b  by RWH (HypC 11) 0
. . . . . . . | | . |
. . . . . . . | | . ⊢  f1(<1of(a),2of(a)>) = b
. . . . . . . | | . | by Assert (<1of(a),2of(a)> = a)
. . . . . . . | | . |
. . . . . . . | | . |\
. . . . . . . | | . | .....assertion .....
. . . . . . . | | . | ⊢  <1of(a),2of(a)> = a
. . . . . . . | | . |   by RWH
. . . . . . . | | . |      (LemmaC
. . . . . . . | | . |       Thm* ∀B:(A→Type), p:a:A×B(a).
. . . . . . . | | . |             <1of(p),2of(p)> = p ∈ a:A×B(a)
. . . . . . . | | . |       {pair_eta_rw})
. . . . . . . | | . |      0
. . . . . . . | | . |
. . . . . . . | | . .\
. . . . . . . | | . . 14. <1of(a),2of(a)> = a
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. . . . . . . | | . . ⊢   by HypSubst 14 0
. . . . . . . | |
. . . . . . . | .\
. . . . . . . | . .....falsecase .....
. . . . . . . | . 12. ¬m-1 = m-1
. . . . . . . | . ⊢  f1
. . . . . . . | .    (<a@0(m-1),f(λz.if z=₂m-1→ 1of(a) else a@0(z) fi)>)
. . . . . . . | .    =
. . . . . . . | .    b  by Auto
. . . . . . . |
. . . . . . . .\
. . . . . . . . .....wf .....
. . . . . . . . 12. a1 : ℕm→ℕn
. . . . . . . . ⊢  f1(<a1(m-1),f(a1)>) = b ∈ Prop  by Auto
END
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