```
bool_is_finite  THM

⊢  Fin(𝔹)  by Unfold ‘finite‘ 0
|
⊢  ∃n:ℕ, f:(ℕn→𝔹). Bij(ℕn; 𝔹; f)  by Witness 2
|
⊢  ∃f:(ℕ2→𝔹). Bij(ℕ2; 𝔹; f)
| by Witness λn.if n=₂0→ true₂ else false₂ fi
|
⊢  Bij(ℕ2; 𝔹; λn.if n=₂0→ true₂ else false₂ fi)  by Analyze 0
|
|\
| ⊢  Inj(ℕ2; 𝔹; λn.if n=₂0→ true₂ else false₂ fi)
| | by Unfold ‘inject‘ 0
| |
| ⊢  ∀a1,a2:ℕ2.
| |  (λn.if n=₂0→ true₂ else false₂ fi)(a1)
| |  =
| |  (λn.if n=₂0→ true₂ else false₂ fi)(a2)
| |  ⇒
| |  a1 = a2  by Reduce 0
| |
| ⊢  ∀a1,a2:ℕ2.
| |  if a1=₂0→ true₂ else false₂ fi = if a2=₂0→ true₂ else false₂ fi
| |  ⇒
| |  a1 = a2  by GenUnivCD
| |
| 1. a1 : ℕ2
| 2. a2 : ℕ2
| 3. if a1=₂0→ true₂ else false₂ fi = if a2=₂0→ true₂ else false₂ fi
| ⊢  a1 = a2  by Decide (a1 = 0)
| |
| |\
| | 4. a1 = 0
| | ⊢   by Decide (a2 = 0)
| | |
| | |\
| | | 5. a2 = 0
| | | ⊢   by Auto
| | |
| | .\
| | . 5. ¬a2 = 0
| | . ⊢   by RWH (HypC 4) 3
| | . |
| | . 3. if 0=₂0→ true₂ else false₂ fi
| | . |  =
| | . |  if a2=₂0→ true₂ else false₂ fi
| | . 4. a1 = 0
| | . 5. ¬a2 = 0
| | . ⊢   by Reduce 3
| | . |
| | . 3. true₂ = if a2=₂0→ true₂ else false₂ fi
| | . 4. a1 = 0
| | . 5. ¬a2 = 0
| | . ⊢   by Assert (a2 = 1)
```

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| | . |
| | . 6. a2 = 1
| | . ⊢   by RWH (HypC 6) 3
| | . |
| | . 3. true₂ = if 1=₂0→ true₂ else false₂ fi
| | . 4. a1 = 0
| | . 5. ¬a2 = 0
| | . 6. a2 = 1
| | . ⊢   by Reduce 3
| | . |
| | . 3. true₂ = false₂
| | . 4. a1 = 0
| | . 5. ¬a2 = 0
| | . 6. a2 = 1
| | . ⊢   by Inst Thm* ¬true₂ = false₂    {btrue_neq_bfalse} []
| | . |
| | . 7. ¬true₂ = false₂
| | . ⊢   by Auto
| |
| .\
| . 4. ¬a1 = 0
| . ⊢   by Assert (a1 = 1)
| . |
| . 5. a1 = 1
| . ⊢   by Decide (a2 = 0)
| . |
| . |\
| . | 6. a2 = 0
| . | ⊢   by RWH (HypC 5) 3
| . | |
| . | 3. if 1=₂0→ true₂ else false₂ fi
| . | |  =
| . | |  if a2=₂0→ true₂ else false₂ fi
| . | 4. ¬a1 = 0
| . | 5. a1 = 1
| . | 6. a2 = 0
| . | ⊢   by RWH (HypC 6) 3
| . | |
| . | 3. if 1=₂0→ true₂ else false₂ fi = if 0=₂0→ true₂ else false₂ fi
| . | 4. ¬a1 = 0
| . | 5. a1 = 1
| . | 6. a2 = 0
| . | ⊢   by Reduce 3
| . | |
| . | 3. false₂ = true₂
| . | 4. ¬a1 = 0
| . | 5. a1 = 1
| . | 6. a2 = 0
| . | ⊢   by Inst Thm* ¬true₂ = false₂    {btrue_neq_bfalse} []
| . | |
| . | 7. ¬true₂ = false₂
| . | ⊢   by RWH (HypC 3) 7
| . | |
| . | 7. ¬true₂ = true₂
| . | ⊢   by Analyze 7
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| . |
| . .\
| . . 6. ¬a2 = 0
| . . ⊢   by Auto
|
.\
. ⊢  Surj(ℕ2; 𝔹; λn.if n=₂0→ true₂ else false₂ fi)
. | by Unfold ‘surject‘ 0
. |
. ⊢  ∀b:𝔹. ∃a:ℕ2. (λn.if n=₂0→ true₂ else false₂ fi)(a) = b  by Auto
. |
. 1. b : 𝔹
. ⊢  ∃a:ℕ2. (λn.if n=₂0→ true₂ else false₂ fi)(a) = b
. | by Decide (b = true₂)
. |
. |\
. | 2. b = true₂
. | ⊢   by Witness 0
. | |
. | |\
. | | ⊢  (λn.if n=₂0→ true₂ else false₂ fi)(0) = b  by Reduce 0
. | |
. | .\
. | . 3. a : ℕ2
. | . ⊢  (λn.if n=₂0→ true₂ else false₂ fi)(a) ∈ 𝔹  by Reduce 0
. |
. .\
. . 2. ¬b = true₂
. . ⊢   by Reduce 0 THEN Witness 1
. . |
. . ⊢  if 1=₂0→ true₂ else false₂ fi = b  by Reduce 0
. . |
. . ⊢  false₂ = b
. . | by Inst Thm* ∀b:𝔹. b = true₂ ∨ b = false₂    {bool_cases} [b]
. . |
. . 3. b = true₂ ∨ b = false₂
. . ⊢   by Analyze 3
END
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