```
fin_dec_fin  THM

⊢  ∀n:ℕ, T:Type, B:(T→Prop).
|  (ℕn ∼ T) & (∀t:T. Dec(B(t))) ⇒ (∃m:ℕ. ℕm ∼ {t:T| B(t) })
| by (Unfold ‘one_one_corr‘ 0) THEN (Analyze 0) THEN (NatInd 1)
|    THEN
|    GenRepD
|    THEN
|    ExRepD
|
|\
| 1. T : Type
| 2. B : T→Prop
| 3. f : ℕ0→T
| 4. g : T→ℕ0
| 5. InvFuns(ℕ0; T; f; g)
| 6. ∀t:T. Dec(B(t))
| ⊢  ∃m:ℕ, f:(ℕm→{t:T| B(t) }), g:({t:T| B(t) }→ℕm).
| |  InvFuns(ℕm; {t:T| B(t) }; f; g)  by InstConcl [0;f;g]
| |
| ⊢  InvFuns(ℕ0; {t:T| B(t) }; f; g)
| | by Unfold ‘inv_funs‘ 0 THEN Unfold ‘inv_funs‘ 5 THEN Analyze 0
| |
| 5. g o f = Id
| 6. f o g = Id
| 7. ∀t:T. Dec(B(t))
| ⊢  f o g = Id ∈ {t:T| B(t) }→{t:T| B(t) }  by Ext
| |
| 8. x : {t:T| B(t) }
| ⊢  (f o g)(x) = Id(x) ∈ {t:T| B(t) }
| | by Analyze -1 THEN Assert ((f o g)(x) = x)
| |
| |\
| | .....assertion .....
| | 8. x : T
| | 9. B(x)
| | ⊢  (f o g)(x) = x  by (RWH (HypC 6) 0) THEN (Reduce 0)
| |
| .\
| . 8. x : T
| . 9. B(x)
| . 10. (f o g)(x) = x
| . ⊢   by Reduce -1 THEN Reduce 0 THEN Analyze
| . |
| . .....set predicate .....
| . 10. f(g(x)) = x
| . ⊢  B(f(g(x)))  by RWH (HypC -1) 0
|
.\
. 1. n : ℤ
. 2. 0<n
. 3. ∀T:Type, B:(T→Prop).
. |  (∃f:(ℕ(n-1)→T), g:(T→ℕ(n-1)). InvFuns(ℕ(n-1); T; f; g))
. |  & (∀t:T. Dec(B(t)))
. |  ⇒
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. |
. |  (∃m:ℕ, f:(ℕm→{t:T| B(t) }), g:({t:T| B(t) }→ℕm).
. |   InvFuns(ℕm; {t:T| B(t) }; f; g))
. 4. T : Type
. 5. B : T→Prop
. 6. f : ℕn→T
. 7. g : T→ℕn
. 8. InvFuns(ℕn; T; f; g)
. 9. ∀t:T. Dec(B(t))
. ⊢  ∃m:ℕ, f:(ℕm→{t:T| B(t) }), g:({t:T| B(t) }→ℕm).
. |  InvFuns(ℕm; {t:T| B(t) }; f; g)
. | by (Unfold `inv_funs` 8) THEN GenRepD
. |
. 8. g o f = Id
. 9. f o g = Id
. 10. ∀t:T. Dec(B(t))
. ⊢   by Assert (f ∈ ℕ(n-1)→{t:T| g(t)<n-1 })
. |
. |\
. | .....assertion .....
. | ⊢  f ∈ ℕ(n-1)→{t:T| g(t)<n-1 }  by ExtWith [] [ℕn→T]
. | |
. | 11. x : ℕ(n-1)
. | ⊢  f(x) ∈ {t:T| g(t)<n-1 }  by Analyze
. | |
. | .....set predicate .....
. | ⊢  g(f(x))<n-1
. |   by (RWH add_composeC 0) THEN (HypSubst 8 0) THEN (Reduce 0)
. |
. .\
. . 11. f ∈ ℕ(n-1)→{t:T| g(t)<n-1 }
. . ⊢   by Assert (g ∈ {t:T| g(t)<n-1 }→ℕ(n-1))
. . |
. . |\
. . | .....assertion .....
. . | ⊢  g ∈ {t:T| g(t)<n-1 }→ℕ(n-1)  by ExtWith [] [T→ℕn]
. . | |
. . | 12. x : {t:T| g(t)<n-1 }
. . | ⊢  g(x) ∈ ℕ(n-1)  by Analyze 12
. . |
. . .\
. . . 12. g ∈ {t:T| g(t)<n-1 }→ℕ(n-1)
. . . ⊢   by InstHyp [{t:T| g(t)<n-1 };B] 3
. . . |
. . . |\
. . . | .....wf .....
. . . | ⊢  {t:T| g(t)<n-1 } ∈ Type  by RW GuardC 12

. . . |
. . . |\
. . . | .....wf .....
. . . | ⊢  B ∈ {t:T| g(t)<n-1 }→Prop  by RW GuardC 12
. . . |
. . . |\
. . . | .....antecedent .....
. . . | ⊢  (∃f:(ℕ(n-1)→{t:T| g(t)<n-1 })
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. . . | |    , g@0:({t:T| g(t)<n-1 }→ℕ(n-1)).
. . . | |   InvFuns(ℕ(n-1); {t:T| g(t)<n-1 }; f; g@0))
. . . | |  & (∀t:{t:T| g(t)<n-1 }. Dec(B(t)))  by Analyze 0
. . . | |
. . . | |\
. . . | | ⊢  ∃f:(ℕ(n-1)→{t:T| g(t)<n-1 })
. . . | | |  , g@0:({t:T| g(t)<n-1 }→ℕ(n-1)).
. . . | | |  InvFuns(ℕ(n-1); {t:T| g(t)<n-1 }; f; g@0)
. . . | | | by InstConcl [f;g]
. . . | | |
. . . | | |\
. . . | | | .....wf .....
. . . | | | ⊢  f ∈ ℕ(n-1)→{t:T| g(t)<n-1 }  by Auto
. . . | | |
. . . | | |\
. . . | | | .....wf .....
. . . | | | ⊢  g ∈ {t:T| g(t)<n-1 }→ℕ(n-1)  by Auto
. . . | | |
. . . | | |\
. . . | | | ⊢  InvFuns(ℕ(n-1); {t:T| g(t)<n-1 }; f; g)
. . . | | | | by Unfold `inv_funs` 0
. . . | | | |
. . . | | | |\
. . . | | | | ⊢  g o f = Id  by Ext
. . . | | | | |
. . . | | | | |\
. . . | | | | | 13. x : ℕ(n-1)
. . . | | | | | ⊢  (g o f)(x) = Id(x)  by Reduce 0
. . . | | | | | |
. . . | | | | | ⊢  g(f(x)) = x  by Assert ((g o f)(x) = x)
. . . | | | | | |
. . . | | | | | |\
. . . | | | | | | .....assertion .....
. . . | | | | | | ⊢  (g o f)(x) = x
. . . | | | | | |   by (HypSubst 8 0) THEN (Reduce 0)
. . . | | | | | |
. . . | | | | | .\
. . . | | | | | . 14. (g o f)(x) = x
. . . | | | | | . ⊢   by Reduce 14
. . . | | | | |
. . . | | | | |\
. . . | | | | | .....wf .....
. . . | | | | | ⊢  ℕ(n-1) = ℕ(n-1)  by Auto
. . . | | | | |
. . . | | | | |\
. . . | | | | | .....wf .....
. . . | | | | | ⊢  g o f = g o f  by RW GuardC 12
. . . | | | | | |
. . . | | | | | 12. g ∈ {t:T| g(t)<n-1 }→ℕ(n-1)
. . . | | | | | ⊢  g ∈ {t:T| g(t)<n-1 }→ℕ(n-1)  by Unfold `guard` 12
. . . | | | | |
. . . | | | | .\
. . . | | | | . .....wf .....
. . . | | | | . ⊢  Id = Id  by Auto
. . . | | | |
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. . . | | | .\
. . . | | | . ⊢  f o g = Id  by Ext
. . . | | | . |
. . . | | | . |\
. . . | | | . | 13. x : {t:T| g(t)<n-1 }
. . . | | | . | ⊢  (f o g)(x) = Id(x)  by Assert ((f o g)(x) = x)
. . . | | | . | |
. . . | | | . | |\
. . . | | | . | | .....assertion .....
. . . | | | . | | ⊢  (f o g)(x) = x
. . . | | | . | |   by (HypSubst 9 0) THEN (Reduce 0)
. . . | | | . | |
. . . | | | . | .\
. . . | | | . | . 14. (f o g)(x) = x
. . . | | | . | . ⊢   by (Reduce -1) THEN (Reduce 0)
. . . | | | . | . |
. . . | | | . | . 14. f(g(x)) = x
. . . | | | . | . ⊢  f(g(x)) = x
. . . | | | . | . | by (RW GuardC 12) THEN (Analyze 13) THEN Analyze
. . . | | | . | . |
. . . | | | . | . .....set predicate .....
. . . | | | . | . 12. g ∈ {t:T| g(t)<n-1 }→ℕ(n-1)
. . . | | | . | . 13. x : T
. . . | | | . | . 14. g(x)<n-1
. . . | | | . | . 15. f(g(x)) = x
. . . | | | . | . ⊢  g(f(g(x)))<n-1  by HypSubst 15 0
. . . | | | . |
. . . | | | . |\
. . . | | | . | .....wf .....
. . . | | | . | ⊢  {t:T| g(t)<n-1 } = {t:T| g(t)<n-1 }
. . . | | | . |   by RW GuardC 12
. . . | | | . |
. . . | | | . |\
. . . | | | . | .....wf .....
. . . | | | . | ⊢  f o g = f o g  by Analyze
. . . | | | . | |
. . . | | | . | |\
. . . | | | . | | .....implicit subterm .....
. . . | | | . | | ⊢  {t:T| g(t)<n-1 } ∈ Type  by RW GuardC 12
. . . | | | . | |
. . . | | | . | |\
. . . | | | . | | .....implicit subterm .....
. . . | | | . | | ⊢  ℕ(n-1) ∈ Type  by Auto
. . . | | | . | |
. . . | | | . | |\
. . . | | | . | | .....implicit subterm .....
. . . | | | . | | ⊢  {t:T| g(t)<n-1 } ∈ Type  by RW GuardC 12
. . . | | | . | |
. . . | | | . | |\
. . . | | | . | | ⊢  f ∈ ℕ(n-1)→{t:T| g(t)<n-1 }  by Auto
. . . | | | . | |
. . . | | | . | .\
. . . | | | . | . ⊢  g ∈ {t:T| g(t)<n-1 }→ℕ(n-1)  by Auto
. . . | | | . |
. . . | | | . .\
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. . . | | | . . .....wf .....
. . . | | | . . ⊢  Id = Id  by RW GuardC 12
. . . | | |
. . . | | |\
. . . | | | .....wf .....
. . . | | | 13. g@0 : {t:T| g(t)<n-1 }→ℕ(n-1)
. . . | | | ⊢  InvFuns(ℕ(n-1); {t:T| g(t)<n-1 }; f; g@0) ∈ Prop
. . . | | |   by RW GuardC 12
. . . | | |
. . . | | .\
. . . | | . .....wf .....
. . . | | . 13. f1 : ℕ(n-1)→{t:T| g(t)<n-1 }
. . . | | . ⊢  (∃g@0:({t:T| g(t)<n-1 }→ℕ(n-1)).
. . . | | .     InvFuns(ℕ(n-1); {t:T| g(t)<n-1 }; f1; g@0))
. . . | | .    ∈ Prop  by RW GuardC 12
. . . | |
. . . | .\
. . . | . ⊢  ∀t:{t:T| g(t)<n-1 }. Dec(B(t))  by RW GuardC 12
. . . |
. . . .\
. . . . 13. ∃m:ℕ, f:(ℕm→{t:{t:T| g(t)<n-1 }| B(t) })
. . . . |    , g@0:({t:{t:T| g(t)<n-1 }| B(t) }→ℕm).
. . . . |    InvFuns(ℕm; {t:{t:T| g(t)<n-1 }| B(t) }; f; g@0)
. . . . ⊢   by (RW GuardC 11) THEN (RW GuardC 12) THEN ExRepD
. . . . |
. . . . 11. f ∈ ℕ(n-1)→{t:T| g(t)<n-1 }
. . . . 12. g ∈ {t:T| g(t)<n-1 }→ℕ(n-1)
. . . . 13. m : ℕ
. . . . 14. f1 : ℕm→{t:{t:T| g(t)<n-1 }| B(t) }
. . . . 15. g@0 : {t:{t:T| g(t)<n-1 }| B(t) }→ℕm
. . . . 16. InvFuns(ℕm; {t:{t:T| g(t)<n-1 }| B(t) }; f1; g@0)
. . . . ⊢   by Decide (B(f(n-1)))
. . . . |
. . . . |\
. . . . | 17. B(f(n-1))
. . . . | ⊢
. . . . | | by (Let (f2 = (λx.if x=₂m→ f(n-1) else f1(x) fi)))
. . . . | |    THEN
. . . . | |    (Let (g2 = (λx.if g(x)=₂n-1→ m else g@0(x) fi)))
. . . . | |
. . . . | |\
. . . . | | .....wf .....
. . . . | | ⊢  (λx.if x=₂m→ f(n-1) else f1(x) fi)
. . . . | | |  ∈ ℕ(m+1)→{t:T| B(t) }  by Analyze
. . . . | | |
. . . . | | 18. x : ℕ(m+1)
. . . . | | ⊢  if x=₂m→ f(n-1) else f1(x) fi ∈ {t:T| B(t) }
. . . . | | | by SplitOnConclITEs
. . . . | | |
. . . . | | |\
. . . . | | | .....truecase .....
. . . . | | | 19. x = m
. . . . | | | ⊢  f(n-1) ∈ {t:T| B(t) }  by Analyze
. . . . | | |
. . . . | | .\
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. . . . | | . .....falsecase .....
. . . . | | . 19. ¬x = m
. . . . | | . ⊢  f1(x) ∈ {t:T| B(t) }
. . . . | | . | by Assert (f1(x) ∈ {t:{t:T| g(t)<n-1 }| B(t) })
. . . . | | . |
. . . . | | . 20. f1(x) ∈ {t:{t:T| g(t)<n-1 }| B(t) }
. . . . | | . ⊢   by (Analyze 20) THEN Analyze
. . . . | |
. . . . | |\
. . . . | | .....wf .....
. . . . | | ⊢  ℕ(m+1)→{t:T| B(t) } ∈ Type  by Auto
. . . . | |
. . . . | |\
. . . . | | .....wf .....
. . . . | | 18. f2 : ℕ(m+1)→{t:T| B(t) }
. . . . | | 19. f2 = (λx.if x=₂m→ f(n-1) else f1(x) fi)
. . . . | | ⊢  (λx.if g(x)=₂n-1→ m else g@0(x) fi)
. . . . | | |  ∈ {t:T| B(t) }→ℕ(m+1)  by Analyze
. . . . | | |
. . . . | | 20. x : {t:T| B(t) }
. . . . | | ⊢  if g(x)=₂n-1→ m else g@0(x) fi ∈ ℕ(m+1)
. . . . | | | by SplitOnConclITEs
. . . . | | |
. . . . | | 21. ¬g(x) = n-1
. . . . | | ⊢  x ∈ {t:{t:T| g(t)<n-1 }| B(t) }
. . . . | | | by (Analyze 20) THEN Analyze THEN Analyze
. . . . | | |

. . . . | | .....set predicate .....
. . . . | | 20. x : T
. . . . | | 21. B(x)
. . . . | | 22. ¬g(x) = n-1
. . . . | | ⊢  g(x)<n-1  by Auto
. . . . | |
. . . . | |\
. . . . | | .....wf .....
. . . . | | 18. f2 : ℕ(m+1)→{t:T| B(t) }
. . . . | | 19. f2 = (λx.if x=₂m→ f(n-1) else f1(x) fi)
. . . . | | ⊢  {t:T| B(t) }→ℕ(m+1) ∈ Type  by Auto
. . . . | |
. . . . | .\
. . . . | . 18. f2 : ℕ(m+1)→{t:T| B(t) }
. . . . | . 19. f2 = (λx.if x=₂m→ f(n-1) else f1(x) fi)
. . . . | . 20. g2 : {t:T| B(t) }→ℕ(m+1)
. . . . | . 21. g2 = (λx.if g(x)=₂n-1→ m else g@0(x) fi)
. . . . | . ⊢   by InstConcl [m+1;f2;g2]
. . . . | . |
. . . . | . ⊢  InvFuns(ℕ(m+1); {t:T| B(t) }; f2; g2)
. . . . | . | by (Unfold ‘inv_funs‘ 16) THEN (Unfold ‘inv_funs‘ 0)
. . . . | . |    THEN
. . . . | . |    GenRepD
. . . . | . |
. . . . | . |\
. . . . | . | 16. g@0 o f1 = Id
. . . . | . | 17. f1 o g@0 = Id
. . . . | . | 18. B(f(n-1))
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. . . . | . | 19. f2 : ℕ(m+1)→{t:T| B(t) }
. . . . | . | 20. f2 = (λx.if x=_2m→ f(n-1) else f1(x) fi)
. . . . | . | 21. g2 : {t:T| B(t) }→ℕ(m+1)
. . . . | . | 22. g2 = (λx.if g(x)=_2n-1→ m else g@0(x) fi)
. . . . | . | ⊢  g2 o f2 = Id  by Ext
. . . . | . | |
. . . . | . | 23. x : ℕ(m+1)
. . . . | . | ⊢  (g2 o f2)(x) = Id(x)
. . . . | . | | by (RWO "20 22" 0) THEN (Reduce 0)
. . . . | . | |
. . . . | . | ⊢  if g(if x=_2m→ f(n-1) else f1(x) fi)=_2n-1→ m
. . . . | . | |  else g@0(if x=_2m→ f(n-1) else f1(x) fi) fi
. . . . | . | |  =
. . . . | . | |  x
. . . . | . | | by ((SplitOnNthConclITE 2) THEN SplitOnConclITE)
. . . . | . | |    THENA
. . . . | . | |    Try (Complete Auto)
. . . . | . | |
. . . . | . | |\
. . . . | . | | .....truecase .....
. . . . | . | | 24. x = m
. . . . | . | | 25. g(f(n-1)) = n-1
. . . . | . | | ⊢  m = x  by Auto
. . . . | . | |
. . . . | . | |\
. . . . | . | | .....falsecase .....
. . . . | . | | 24. x = m
. . . . | . | | 25. ¬g(f(n-1)) = n-1
. . . . | . | | ⊢  g@0(f(n-1)) = x
. . . . | . | |   by (RWH add_composeC 25) THEN (HypSubst 8 25)
. . . . | . | |      THEN
. . . . | . | |      (Reduce 25)
. . . . | . | |
. . . . | . | |\
. . . . | . | | .....truecase .....
. . . . | . | | 24. ¬x = m
. . . . | . | | 25. g(f1(x)) = n-1
. . . . | . | | ⊢  m = x
. . . . | . | | | by Assert (f1(x) ∈ {t:{t:T| g(t)<n-1 }| B(t) })
. . . . | . | | |
. . . . | . | | 26. f1(x) ∈ {t:{t:T| g(t)<n-1 }| B(t) }
. . . . | . | | ⊢   by (Analyze 26) THEN (Analyze 26)
. . . . | . | |
. . . . | . | .\
. . . . | . | . .....falsecase .....
. . . . | . | . 24. ¬x = m
. . . . | . | . 25. ¬g(f1(x)) = n-1
. . . . | . | . ⊢  g@0(f1(x)) = x  by Assert ((g@0 o f1)(x) = x)
. . . . | . | . |
. . . . | . | . |\
. . . . | . | . | .....assertion .....
. . . . | . | . | ⊢  (g@0 o f1)(x) = x
. . . . | . | . |   by (HypSubst 16 0) THEN (Reduce 0)
. . . . | . | . |
. . . . | . | . .\
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. . . . | . | . . 26. (g@0 o f1)(x) = x
. . . . | . | . . ⊢   by Reduce 26
. . . . | . |
. . . . | . .\
. . . . | . . 16. g@0 o f1 = Id
. . . . | . . 17. f1 o g@0 = Id
. . . . | . . 18. B(f(n-1))
. . . . | . . 19. f2 : ℕ(m+1)→{t:T| B(t) }
. . . . | . . 20. f2 = (λx.if x=_2m→ f(n-1) else f1(x) fi)
. . . . | . . 21. g2 : {t:T| B(t) }→ℕ(m+1)
. . . . | . . 22. g2 = (λx.if g(x)=_2n-1→ m else g@0(x) fi)
. . . . | . . ⊢  f2 o g2 = Id
. . . . | . . | by Ext THEN (RWO "20 22" 0) THEN (Reduce 0)
. . . . | . . |
. . . . | . . 23. x : {t:T| B(t) }
. . . . | . . ⊢  if if g(x)=_2n-1→ m else g@0(x) fi=_2m→ f(n-1)
. . . . | . . |  else f1(if g(x)=_2n-1→ m else g@0(x) fi) fi
. . . . | . . |  =
. . . . | . . |  x  by SplitOnNthConclITE 2 THENA Try (Complete Auto)
. . . . | . . |
. . . . | . . |\
. . . . | . . | .....truecase .....
. . . . | . . | 24. g(x) = n-1
. . . . | . . | ⊢  if m=_2m→ f(n-1) else f1(m) fi = x
. . . . | . . | | by SplitOnConclITE THENA Try (Complete Auto)
. . . . | . . | |
. . . . | . . | |\
. . . . | . . | | .....truecase .....
. . . . | . . | | 25. m = m
. . . . | . . | | ⊢  f(n-1) = x
. . . . | . . | | | by (Assert (n-1 = g(x)))
. . . . | . . | | |    THEN
. . . . | . . | | |    (ApplyFunToHypEquands f -1)
. . . . | . . | | |
. . . . | . . | | 26. n-1 = g(x)
. . . . | . . | | 27. f(n-1) = f(g(x))
. . . . | . . | | ⊢
. . . . | . . | | | by (RWH add_composeC 27) THEN (HypSubst 9 27)
. . . . | . . | | |    THEN
. . . . | . . | | |    (Reduce 27)
. . . . | . . | | |
. . . . | . . | | 27. f(n-1) = x
. . . . | . . | | ⊢   by Analyze
. . . . | . . | |
. . . . | . . | .\
. . . . | . . | . .....falsecase .....
. . . . | . . | . 25. ¬m = m
. . . . | . . | . ⊢  f1(m) = x  by Auto
. . . . | . . |
. . . . | . . .\
. . . . | . . . .....falsecase .....
. . . . | . . . 24. ¬g(x) = n-1
. . . . | . . . ⊢  if g@0(x)=_2m→ f(n-1) else f1(g@0(x)) fi = x
. . . . | . . . | by Assert (x ∈ {t:{t:T| g(t)<n-1 }| B(t) })
. . . . | . . . |
```

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. . . . | . . . |\
. . . . | . . . | .....assertion .....
. . . . | . . . | ⊢  x ∈ {t:{t:T| g(t)<n-1 }| B(t) }
. . . . | . . . |   by (Analyze 23) THEN Analyze THEN Analyze
. . . . | . . . |
. . . . | . . . .\
. . . . | . . . . 25. x ∈ {t:{t:T| g(t)<n-1 }| B(t) }
. . . . | . . . . ⊢   by SplitOnConclITE THENA Try (Complete Auto)
. . . . | . . . . |
. . . . | . . . . |\
. . . . | . . . . | .....truecase .....
. . . . | . . . . | 26. g@0(x) = m
. . . . | . . . . | ⊢  f(n-1) = x  by Assert (g@0(x) ∈ ℕm)
. . . . | . . . . |
. . . . | . . . . .\
. . . . | . . . . . .....falsecase .....
. . . . | . . . . . 26. ¬g@0(x) = m
. . . . | . . . . . ⊢  f1(g@0(x)) = x  by Assert ((f1 o g@0)(x) = x)
. . . . | . . . . . |
. . . . | . . . . . |\
. . . . | . . . . . | .....assertion .....
. . . . | . . . . . | ⊢  (f1 o g@0)(x) = x
. . . . | . . . . . |   by (HypSubst 17 0) THEN (Reduce 0)
. . . . | . . . . . |
. . . . | . . . . . .\
. . . . | . . . . . . 27. (f1 o g@0)(x) = x
. . . . | . . . . . . ⊢   by SwapEquands 0 THEN Analyze
. . . . | . . . . . . |
. . . . | . . . . . . |\
. . . . | . . . . . . | ⊢  x = f1(g@0(x))
. . . . | . . . . . . |   by Reduce 27 THEN Analyze 27
. . . . | . . . . . . |      THEN
. . . . | . . . . . . |      Analyze 27
. . . . | . . . . . . |
. . . . | . . . . . . .\
. . . . | . . . . . . . .....set predicate .....
. . . . | . . . . . . . ⊢  B(x)  by Analyze 23 THEN HUnhide
. . . . |
. . . . .\
. . . . . 17. ¬B(f(n-1))
. . . . . ⊢
. . . . . | by (Assert (f1 ∈ ℕm→{t:T| B(t) }))
. . . . . |    THEN
. . . . . |    (Assert (g@0 ∈ {t:T| B(t) }→ℕm))
. . . . . |
. . . . . |\
. . . . . | .....assertion .....
. . . . . | 18. f1 ∈ ℕm→{t:T| B(t) }
. . . . . | ⊢  g@0 ∈ {t:T| B(t) }→ℕm
. . . . . | | by ExtWith [] [{t:{t:T| g(t)<n-1 }| B(t) }→ℕm]
. . . . . | |
. . . . . | 19. x : {t:T| B(t) }
. . . . . | ⊢  x ∈ {t:{t:T| g(t)<n-1 }| B(t) }
. . . . . | | by (Analyze 18 THEN Analyze) THEN Analyze
. . . . . | |
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. . . . . . | |\
. . . . . . | | .....set predicate .....
. . . . . . | | 18. f1 = f1
. . . . . . | | 19. x : {t:T| B(t) }
. . . . . . | | ⊢  g(x)<n-1  by Assert (¬g(x) = n-1)
. . . . . . | | |
. . . . . . | | |\
. . . . . . | | | .....assertion .....
. . . . . . | | | ⊢  ¬g(x) = n-1
. . . . . . | | | | by (Analyze 0) THEN (RWH (RevHypC -1) 17)
. . . . . . | | | |
. . . . . . | | | 17. f1 = f1
. . . . . . | | | 18. x : {t:T| B(t) }
. . . . . . | | | 19. g(x) = n-1
. . . . . . | | | 20. ¬B(f(g(x)))
. . . . . . | | | ⊢  False
. . . . . . | | | | by (RWH (AddrC [1;2] add_composeC) -1
. . . . . . | | | |      THEN
. . . . . . | | | |      RWH (HypC 9) -1)
. . . . . . | | | |     THEN
. . . . . . | | | |     (Reduce -1)
. . . . . . | | | |
. . . . . . | | | 20. ¬B(x)
. . . . . . | | | ⊢   by Analyze 18
. . . . . . | | |
. . . . . . | | .\
. . . . . . | | . 20. ¬g(x) = n-1
. . . . . . | | . ⊢   by Auto
. . . . . . | |
. . . . . . | .\
. . . . . . | . .....set predicate .....
. . . . . . | . 18. f1 = f1
. . . . . . | . 19. x : {t:T| B(t) }
. . . . . . | . ⊢  B(x)  by Analyze -1 THEN HUnhide
. . . . . . |
. . . . . . .\
. . . . . . . 18. f1 ∈ ℕm→{t:T| B(t) }
. . . . . . . 19. g@0 ∈ {t:T| B(t) }→ℕm
. . . . . . . ⊢   by InstConcl [m;f1;g@0]
. . . . . . . |
. . . . . . . ⊢  InvFuns(ℕm; {t:T| B(t) }; f1; g@0)
. . . . . . . | by (Analyze 16) THEN (Analyze 0)
. . . . . . . |
. . . . . . . 16. g@0 o f1 = Id
. . . . . . . 17. f1 o g@0 = Id
. . . . . . . 18. ¬B(f(n-1))
. . . . . . . 19. f1 ∈ ℕm→{t:T| B(t) }
. . . . . . . 20. g@0 ∈ {t:T| B(t) }→ℕm
. . . . . . . ⊢  f1 o g@0 = Id  by Ext THEN (HypSubst 17 0)
. . . . . . . |
. . . . . . . |\
. . . . . . . | 21. x : {t:T| B(t) }
. . . . . . . | ⊢  Id(x) = Id(x)  by Reduce 0
. . . . . . . |
. . . . . . . .\
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. . . . . . . . 21. x : {t:T| B(t) }
. . . . . . . . 22. {t:{t:T| g(t)<n-1 }| B(t) }→{t:{t:T| g(t)<n-1 }
. . . . . . . . |                                  | B(t) }
. . . . . . . . ⊢  x ∈ {t:{t:T| g(t)<n-1 }| B(t) }
. . . . . . . . | by (Analyze -2 THEN Analyze) THEN Analyze
. . . . . . . . |
. . . . . . . . .....set predicate .....
. . . . . . . . 21. x : T
. . . . . . . . 22. B(x)
. . . . . . . . 23. {t:{t:T| g(t)<n-1 }| B(t) }→{t:{t:T| g(t)<n-1 }
. . . . . . . . |                                  | B(t) }
. . . . . . . . ⊢  g(x)<n-1  by Assert (¬g(x) = n-1)
. . . . . . . . |
. . . . . . . . .....assertion .....
. . . . . . . . ⊢  ¬g(x) = n-1
. . . . . . . . | by (Analyze 0) THEN (RWH (RevHypC -1) 17)
. . . . . . . . |
. . . . . . . . 17. ¬B(f(n-1))
. . . . . . . . 18. f1 ∈ ℕm→{t:T| B(t) }
. . . . . . . . 19. g@0 ∈ {t:T| B(t) }→ℕm
. . . . . . . . 20. x : T
. . . . . . . . 21. B(x)
. . . . . . . . 22. {t:{t:T| g(t)<n-1 }| B(t) }→{t:{t:T| g(t)<n-1 }
. . . . . . . . |                                  | B(t) }
. . . . . . . . 23. g(x) = n-1
. . . . . . . . 24. f1 o g@0 = Id
. . . . . . . . ⊢  False  by RWH (RevHypC -2) 17
. . . . . . . . |
. . . . . . . . 17. f1 ∈ ℕm→{t:T| B(t) }
. . . . . . . . 18. g@0 ∈ {t:T| B(t) }→ℕm
. . . . . . . . 19. x : T
. . . . . . . . 20. B(x)
. . . . . . . . 21. {t:{t:T| g(t)<n-1 }| B(t) }→{t:{t:T| g(t)<n-1 }
. . . . . . . .                                    | B(t) }
. . . . . . . . 22. g(x) = n-1
. . . . . . . . 23. f1 o g@0 = Id
. . . . . . . . 24. ¬B(f(g(x)))
. . . . . . . . ⊢
. . . . . . . .   by (RWH (AddrC [1;2] add_composeC) -1
. . . . . . . .       THEN
. . . . . . . .       RWH (HypC 9) -1)
. . . . . . . .      THEN
. . . . . . . .      (Reduce -1)
END
```