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inv_of_fin_is_fin  THM

⊢  ∀f:(T→S).
|  Fin(S) & (∀s:S. Dec(∃t:T. f(t) = s)) ⇒ Fin(x,y:T//(f(x) = f(y)))
| by UnivCD
|
1. T : Type
2. S : Type
3. f : T→S
4. Fin(S) & (∀s:S. Dec(∃t:T. f(t) = s))
⊢  Fin(x,y:T//(f(x) = f(y)))  by Analyze 4
|
4. Fin(S)
5. ∀s:S. Dec(∃t:T. f(t) = s)
⊢   by Assert (EquivRel x,y:T. f(x) = f(y))
|
|\
| .....assertion .....
| ⊢  EquivRel x,y:T. f(x) = f(y)
|   by Analyze 0 THEN Analyze 0 THEN Try Auto THEN Analyze 0
|
.\
. 6. EquivRel x,y:T. f(x) = f(y)
. ⊢
. | by Inst Thm* Fin(T) & (T ∼ S) ⇒ Fin(S)    {one_one_preser_fin}
. |    [{s:S| ∃t:T. f(t) = s };x,y:T//(f(x) = f(y))]
. |
. |\
. | ⊢  Fin({s:S| ∃t:T. f(t) = s })
. | | by Inst
. | |    Thm* ∀B:(T→Prop).
. | |         Fin(T) & (∀t:T. Dec(B(t))) ⇒ Fin({t:T| B(t) })
. | |    {finite_decidable_subset}
. | |    [S;λs.∃t:T. f(t) = s]
. | |
. | |\
. | | .....decidable? .....
. | | 7. t : S
. | | ⊢  Dec((λs.∃t:T. f(t) = s)(t))  by Reduce 0
. | |
. | .\
. | . 7. Fin({t:S| (λs.∃t:T. f(t) = s)(t) })
. | . ⊢   by Reduce 7
. |
. |\
. | ⊢  {s:S| ∃t:T. f(t) = s } ∼ (x,y:T//(f(x) = f(y)))
. | | by RWH
. | |    (RevLemmaC
. | |     Thm* (∃f:(A→B). Bij(A; B; f)) ⟺ (A ∼ B)
. | |     {bij_iff_1_1_corr})
. | |    0
. | |
. | ⊢  ∃f1:({s:S| ∃t:T. f(t) = s }→(x,y:T//(f(x) = f(y)))).
. | |  Bij({s:S| ∃t:T. f(t) = s }; x,y:T//(f(x) = f(y)); f1)
. | | by Inst
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. | |    Thm* ∀R:(S→T→Prop).
. | |         (∀s:S. Dec(∃t:T. R(s,t))) ⇒
. | |         (∃f:({s:S| ∃t:T. R(s,t) }→T).
. | |          ∀s:{s:S| ∃t:T. R(s,t) }. R(s,f(s)))
. | |    {set_function}
. | |    [S;T;λs,t. f(t) = s]
. | |
. | |\
. | | .....decidable? .....
. | | 7. s : S
. | | ⊢  Dec(∃t:T. (λs,t. f(t) = s)(s,t))  by Reduce 0
. | |
. | .\
. | . 7. ∃f@0:({s:S| ∃t:T. (λs,t. f(t) = s)(s,t) }→T).
. | . |  ∀s:{s:S| ∃t:T. (λs,t. f(t) = s)(s,t) }.
. | . |  (λs,t. f(t) = s)(s,f@0(s))
. | . ⊢   by Reduce 7
. | . |
. | . 7. ∃f@0:({s:S| ∃t:T. f(t) = s }→T).
. | . |  ∀s:{s:S| ∃t:T. f(t) = s }. f(f@0(s)) = s
. | . ⊢   by Analyze 7
. | . |
. | . 7. f@0 : {s:S| ∃t:T. f(t) = s }→T
. | . 8. ∀s:{s:S| ∃t:T. f(t) = s }. f(f@0(s)) = s
. | . ⊢   by Witness f@0
. | . |
. | . |\
. | . | .....wf .....
. | . | ⊢  f@0 ∈ {s:S| ∃t:T. f(t) = s }→(x,y:T//(f(x) = f(y)))
. | . | | by Unfold ‘member‘ 0
. | . | |
. | . | ⊢  f@0 = f@0  by Ext
. | . | |
. | . | |\
. | . | | .....wf .....
. | . | | ⊢   by Auto
. | . | |
. | . | |\
. | . | | 9. x : {s:S| ∃t:T. f(t) = s }
. | . | | ⊢  f@0(x) = f@0(x)  by Auto
. | . | |
. | . | .\
. | . | . .....wf .....
. | . | . ⊢  {s:S| ∃t:T. f(t) = s } = {s:S| ∃t:T. f(t) = s }  by Auto
. | . |
. | . |\
. | . | ⊢  Bij({s:S| ∃t:T. f(t) = s }; x,y:T//(f(x) = f(y)); f@0)
. | . | | by Analyze 0 THEN Analyze 0
. | . | |
. | . | |\
. | . | | 9. a1 : {s:S| ∃t:T. f(t) = s }
. | . | | ⊢  ∀a2:{s:S| ∃t:T. f(t) = s }. f@0(a1) = f@0(a2) ⇒ a1 = a2
. | . | | | by UnivCD
. | . | | |
. | . | | 10. a2 : {s:S| ∃t:T. f(t) = s }
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. | . | | 11. f@0(a1) = f@0(a2)
. | . | | ⊢  a1 = a2  by ApFunToHypEquands ‘x’ (f(x)) S 11
. | . | | |
. | . | | |\
. | . | | | .....fun wf .....
. | . | | | 12. x : x,y:T//(f(x) = f(y))
. | . | | | ⊢  f(x) = f(x)  by Analyze 12
. | . | | | |
. | . | | | 12. x1 : T
. | . | | | 13. x2 : T
. | . | | | 14. f(x1) = f(x2)
. | . | | | ⊢  f(x1) = f(x2)  by Auto
. | . | | |
. | . | | |\
. | . | | | .....wf .....
. | . | | | ⊢  x,y:T//(f(x) = f(y)) = x,y:T//(f(x) = f(y))  by Auto
. | . | | |
. | . | | .\
. | . | | . 12. f(f@0(a1)) = f(f@0(a2))
. | . | | . ⊢   by RWH (HypC 8) 12
. | . | | . |
. | . | | . 12. a1 = a2
. | . | | . ⊢   by Analyze THEN Try Auto
. | . | | . |
. | . | | . .....set predicate .....
. | . | | . ⊢  ∃t:T. f(t) = a1  by Analyze 9
. | . | | . |
. | . | | . 9. a1 : S
. | . | | . 10. ∃t:T. f(t) = a1  [not for witness]
. | . | | . 11. a2 : {s:S| ∃t:T. f(t) = s }
. | . | | . 12. f@0(a1) = f@0(a2)
. | . | | . 13. a1 = a2
. | . | | . ⊢   by Unhide
. | . | |
. | . | .\
. | . | . 9. b : x,y:T//(f(x) = f(y))
. | . | . ⊢  ∃a:{s:S| ∃t:T. f(t) = s }. f@0(a) = b  by Witness f(b)
. | . | . |
. | . | . |\
. | . | . | .....wf .....
. | . | . | ⊢  f(b) ∈ {s:S| ∃t:T. f(t) = s }  by Analyze
. | . | . | |
. | . | . | |\
. | . | . | | ⊢  f(b) ∈ S  by Unfold ‘member‘ 0
. | . | . | | |
. | . | . | | ⊢  f(b) = f(b)  by Analyze 9
. | . | . | | |
. | . | . | | |\
. | . | . | | | .....wf .....
. | . | . | | | 9. b1 : T
. | . | . | | | 10. b2 : T
. | . | . | | | ⊢  f(b1) = f(b2) ∈ Type  by Auto
. | . | . | | |
. | . | . | | |\
. | . | . | | | .....wf .....
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. | . | . | | | 8. ∀s:{s:S| ∃t:T. f(t) = s }. f(f@0(s)) = s
. | . | . | | | ⊢  S ∈ Type  by Auto
. | . | . | | |
. | . | . | | .\
. | . | . | | . 9. b1 : T
. | . | . | | . 10. b2 : T
. | . | . | | . 11. f(b1) = f(b2)
. | . | . | | . ⊢  f(b1) = f(b2)  by Auto
. | . | . | |
. | . | . | |\
. | . | . | | .....set predicate .....
. | . | . | | ⊢  ∃t:T. f(t) = f(b)  by CSquash
. | . | . | | |
. | . | . | | |\
. | . | . | | | .....wf .....
. | . | . | | | ⊢  (∃t:T. f(t) = f(b)) ∈ Prop  by Analyze
. | . | . | | | |
. | . | . | | | |\
. | . | . | | | | ⊢  T ∈ Type  by Auto
. | . | . | | | |
. | . | . | | | .\
. | . | . | | | . 10. t : T
. | . | . | | | . ⊢  f(t) = f(b) ∈ Prop  by Analyze
. | . | . | | | . |
. | . | . | | | . |\
. | . | . | | | . | ⊢  S ∈ Type  by Auto
. | . | . | | | . |
. | . | . | | | . |\
. | . | . | | | . | ⊢  f(t) ∈ S  by Auto
. | . | . | | | . |
. | . | . | | | . .\
. | . | . | | | . . ⊢  f(b) ∈ S  by Unfold ‘member‘ 0
. | . | . | | | . . |
. | . | . | | | . . ⊢  f(b) = f(b)  by Analyze 9
. | . | . | | |
. | . | . | | |\
. | . | . | | | .....wf .....
. | . | . | | | ⊢  f(b) ∈ S  by Unfold ‘member‘ 0
. | . | . | | | |
. | . | . | | | ⊢  f(b) = f(b)  by Analyze 9
. | . | . | | |
. | . | . | | .\
. | . | . | | . ⊢  ↓(∃t:T. f(t) = f(b))  by UseEqWitness *
. | . | . | | . |
. | . | . | | . ⊢  * = *  by Analyze 9
. | . | . | | . |
. | . | . | | . |\
. | . | . | | . | .....wf .....
. | . | . | | . | 9. b1 : T
. | . | . | | . | 10. b2 : T
. | . | . | | . | ⊢  f(b1) = f(b2) ∈ Type  by Auto
. | . | . | | . |
. | . | . | | . |\
. | . | . | | . | .....wf .....
. | . | . | | . | ⊢  ↓(∃t:T. f(t) = f(b)) ∈ Type  by Analyze
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. | . | . | | . | |
. | . | . | | . | ⊢  (∃t:T. f(t) = f(b)) ∈ Prop  by Analyze
. | . | . | | . | |
. | . | . | | . | |\
. | . | . | | . | | ⊢  T ∈ Type  by Auto
. | . | . | | . | |
. | . | . | | . | .\
. | . | . | | . | . 10. t : T
. | . | . | | . | . ⊢  f(t) = f(b) ∈ Prop  by Analyze
. | . | . | | . | . |
. | . | . | | . | . |\
. | . | . | | . | . | ⊢  S ∈ Type  by Auto
. | . | . | | . | . |
. | . | . | | . | . |\
. | . | . | | . | . | ⊢  f(t) ∈ S  by Auto
. | . | . | | . | . |
. | . | . | | . | . .\
. | . | . | | . | . . ⊢  f(b) ∈ S  by Unfold ‘member‘ 0
. | . | . | | . | . . |
. | . | . | | . | . . ⊢  f(b) = f(b)  by Analyze 9
. | . | . | | . |
. | . | . | | . .\
. | . | . | | . . 9. b1 : T
. | . | . | | . . 10. b2 : T
. | . | . | | . . 11. f(b1) = f(b2)
. | . | . | | . . ⊢  * = *  by Analyze
. | . | . | | . . |
. | . | . | | . . ⊢  ∃t:T. f(t) = f(b1)  by Witness b1
. | . | . | |
. | . | . | .\
. | . | . | . .....wf .....
. | . | . | . 10. s : S
. | . | . | . ⊢  (∃t:T. f(t) = s) ∈ Type  by Auto
. | . | . |
. | . | . |\
. | . | . | ⊢  f@0(f(b)) = b  by Analyze 9
. | . | . | |
. | . | . | 9. b1 : T
. | . | . | 10. b2 : T
. | . | . | 11. f(b1) = f(b2)
. | . | . | ⊢  f@0(f(b1)) = b2  by Analyze
. | . | . | |
. | . | . | |\
. | . | . | | ⊢  f(b1) ∈ {s:S| ∃t:T. f(t) = s }  by Analyze
. | . | . | | |
. | . | . | | |\
. | . | . | | | ⊢  f(b1) ∈ S  by Auto
. | . | . | | |
. | . | . | | .\
. | . | . | | . .....set predicate .....
. | . | . | | . ⊢  ∃t:T. f(t) = f(b1)  by Witness b1
. | . | . | |
. | . | . | .\
. | . | . | . ⊢  f(f@0(f(b1))) = f(b2)  by RWH (HypC 8) 0
. | . | . | . |
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. | . | . | . .....wf .....
. | . | . | . ⊢  f(b1) ∈ {s:S| ∃t:T. f(t) = s }
. | . | . | . | by Analyze THEN Try Auto
. | . | . | . |
. | . | . | . .....set predicate .....
. | . | . | . ⊢  ∃t:T. f(t) = f(b1)  by Witness b1
. | . | . |
. | . | . .\
. | . | . . .....wf .....
. | . | . . 10. a : {s:S| ∃t:T. f(t) = s }
. | . | . . ⊢  f@0(a) = b ∈ Prop  by Auto
. | . |
. | . .\
. | . . .....wf .....
. | . . 9. f1 : {s:S| ∃t:T. f(t) = s }→(x,y:T//(f(x) = f(y)))
. | . . ⊢  Bij({s:S| ∃t:T. f(t) = s }; x,y:T//(f(x) = f(y)); f1)
. | . .    ∈ Prop  by Auto
. |
. .\
. . 7. Fin(x,y:T//(f(x) = f(y)))
. . ⊢   by Auto
END
```