```
one2one_preserves_fin  THM

⊢  (T ∼ U) & Fin(T) ⇒ Fin(U)  by UnivCD
|
1. T : Type
2. U : Type
3. (T ∼ U) & Fin(T)
⊢  Fin(U)  by Analyze 3
|
3. T ∼ U
4. Fin(T)
⊢   by Analyze 3
|
3. f : T→U
4. ∃g:(U→T). InvFuns(T; U; f; g)
5. Fin(T)
⊢   by Analyze 4
|
4. g : U→T
5. InvFuns(T; U; f; g)
6. Fin(T)
⊢   by Analyze 5
|
5. g o f = Id
6. f o g = Id
7. Fin(T)
⊢   by Analyze 7
|
7. n : ℕ
8. ∃f:(ℕn→T). Bij(ℕn; T; f)
⊢   by Analyze 8
|
8. f1 : ℕn→T
9. Bij(ℕn; T; f1)
⊢   by Unfold ‘finite‘ 0
|
⊢  ∃n:ℕ, f:(ℕn→U). Bij(ℕn; U; f)  by Witness n
|
⊢  ∃f:(ℕn→U). Bij(ℕn; U; f)  by Witness f o f1
|
⊢  Bij(ℕn; U; f o f1)  by Unfold ‘biject‘ 9
|
9. Inj(ℕn; T; f1) & Surj(ℕn; T; f1)
⊢   by Analyze 9
|
9. Inj(ℕn; T; f1)
10. Surj(ℕn; T; f1)
⊢   by Unfold ‘biject‘ 0
|
⊢  Inj(ℕn; U; f o f1) & Surj(ℕn; U; f o f1)  by Analyze 0
|
|\
| ⊢  Inj(ℕn; U; f o f1)  by Unfold ‘inject‘ 0
| |
| ⊢  ∀a1,a2:ℕn. (f o f1)(a1) = (f o f1)(a2) ⇒ a1 = a2  by UnivCD
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| |
| 11. a1 : ℕn
| 12. a2 : ℕn
| 13. (f o f1)(a1) = (f o f1)(a2)
| ⊢  a1 = a2  by ApFunToHypEquands ‘x’ (g(x)) T 13
| |
| 14. g((f o f1)(a1)) = g((f o f1)(a2))
| ⊢   by RWH add_composeC 14
| |
| 14. (g o f o f1)(a1) = (g o f o f1)(a2)
| ⊢
| | by RWH
| |    (LemmaC
| |     Thm* ∀f:(A→B), g:(B→C), h:(C→D).
| |          h o (g o f) = (h o g) o f ∈ A→D
| |     {comp_assoc})
| |    14
| |
| 14. (g o f o f1)(a1) = (g o f o f1)(a2)
| ⊢   by RWH (HypC 5) 14
| |
| 14. (Id o f1)(a1) = (Id o f1)(a2)
| ⊢   by Reduce 14
| |
| 14. f1(a1) = f1(a2)
| ⊢   by Unfold ‘inject‘ 9
| |
| 9. ∀a1,a2:ℕn. f1(a1) = f1(a2) ⇒ a1 = a2
| 10. Surj(ℕn; T; f1)
| 11. a1 : ℕn
| 12. a2 : ℕn
| 13. (f o f1)(a1) = (f o f1)(a2)
| 14. f1(a1) = f1(a2)
| ⊢   by InstHyp [a1;a2] 9
|
.\
. ⊢  Surj(ℕn; U; f o f1)  by Unfold ‘surject‘ 0
. |
. ⊢  ∀b:U. ∃a:ℕn. (f o f1)(a) = b  by UnivCD
. |
. 11. b : U
. ⊢  ∃a:ℕn. (f o f1)(a) = b  by Unfold ‘surject‘ 10
. |
. 10. ∀b:T. ∃a:ℕn. f1(a) = b
. 11. b : U
. ⊢   by Witness10 g(b)
. |

. 10. b : U
. 11. ∃a:ℕn. f1(a) = g(b)
. ⊢   by Analyze 11
. |
. 11. a : ℕn
. 12. f1(a) = g(b)
. ⊢   by ApFunToHypEquands ‘x’ (f(x)) U 12
. |
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. 13. f(f1(a)) = f(g(b))
. ⊢   by RWH add_composeC 13
. |
. 13. (f o f1)(a) = (f o g)(b)
. ⊢   by RWH (HypC 6) 13
. |
. 13. (f o f1)(a) = Id(b)
. ⊢   by Reduce 13
. |
. 13. f(f1(a)) = b
. ⊢   by Witness a
. |
. ⊢  (f o f1)(a) = b  by Reduce 0
END
```