```
one_one_preser_fin  THM

⊢  Fin(T) & (T ∼ S) ⇒ Fin(S)
| by Unfold ‘finite‘ 0 THEN Unfold ‘one_one_corr‘ 0 THEN UnivCD
|
1. T : Type
2. S : Type
3. (∃n:ℕ, f:(ℕn→T). Bij(ℕn; T; f))
|  & (∃f:(T→S), g:(S→T). InvFuns(T; S; f; g))
⊢  ∃n:ℕ, f:(ℕn→S). Bij(ℕn; S; f)
| by Analyze 3 THEN Analyze 3 THEN Analyze 4
|
3. n : ℕ
4. f : ℕn→T
5. Bij(ℕn; T; f)
6. ∃f:(T→S), g:(S→T). InvFuns(T; S; f; g)
⊢   by Analyze 6 THEN Analyze 7
|
6. f1 : T→S
7. g : S→T
8. InvFuns(T; S; f1; g)
⊢   by Analyze 8
|
8. g o f1 = Id
9. f1 o g = Id
⊢   by Witness n
|
⊢  ∃f:(ℕn→S). Bij(ℕn; S; f)  by Witness f1 o f
|
⊢  Bij(ℕn; S; f1 o f)  by Analyze 0 THEN Analyze 0
|
|\
| 10. a1 : ℕn
| ⊢  ∀a2:ℕn. (f1 o f)(a1) = (f1 o f)(a2) ⇒ a1 = a2  by UnivCD
| |
| 11. a2 : ℕn
| 12. (f1 o f)(a1) = (f1 o f)(a2)
| ⊢  a1 = a2  by ApFunToHypEquands ‘x’ (g(x)) T 12
| |
| 13. g((f1 o f)(a1)) = g((f1 o f)(a2))
| ⊢   by RWH add_composeC 13
| |
| 13. (g o f1 o f)(a1) = (g o f1 o f)(a2)
| ⊢
| | by RWH
| |    (LemmaC
| |     Thm* ∀f:(A→B), g:(B→C), h:(C→D).
| |          h o (g o f) = (h o g) o f ∈ A→D
| |     {comp_assoc})
| |    13
| |
| 13. (g o f1 o f)(a1) = (g o f1 o f)(a2)
| ⊢   by RWH (HypC 8) 13
| |
| 13. (Id o f)(a1) = (Id o f)(a2)
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| ⊢   by Reduce 13
| |
| 13. f(a1) = f(a2)
| ⊢   by Analyze 5
| |
| 5. Inj(ℕn; T; f)
| 6. Surj(ℕn; T; f)
| 7. f1 : T→S
| 8. g : S→T
| 9. g o f1 = Id
| 10. f1 o g = Id
| 11. a1 : ℕn
| 12. a2 : ℕn
| 13. (f1 o f)(a1) = (f1 o f)(a2)
| 14. f(a1) = f(a2)
| ⊢   by Unfold ‘inject‘ 5
| |
| 5. ∀a1,a2:ℕn. f(a1) = f(a2) ⇒ a1 = a2
| 6. Surj(ℕn; T; f)
| 7. f1 : T→S
| 8. g : S→T
| 9. g o f1 = Id
| 10. f1 o g = Id
| 11. a1 : ℕn
| 12. a2 : ℕn
| 13. (f1 o f)(a1) = (f1 o f)(a2)
| 14. f(a1) = f(a2)
| ⊢   by BackThru 5
|
.\
. 10. b : S
. ⊢  ∃a:ℕn. (f1 o f)(a) = b  by Analyze 5
. |
. 5. Inj(ℕn; T; f)
. 6. Surj(ℕn; T; f)
. 7. f1 : T→S
. 8. g : S→T
. 9. g o f1 = Id
. 10. f1 o g = Id
. 11. b : S
. ⊢   by Unfold ‘surject‘ 6
. |
. 6. ∀b:T. ∃a:ℕn. f(a) = b
. 7. f1 : T→S
. 8. g : S→T
. 9. g o f1 = Id
. 10. f1 o g = Id
. 11. b : S
. ⊢   by Witness6 g(b)
. |
. 6. f1 : T→S
. 7. g : S→T
. 8. g o f1 = Id
. 9. f1 o g = Id
. 10. b : S
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. 11. ∃a:ℕn. f(a) = g(b)
. ⊢   by Analyze 11
. |
. 11. a : ℕn
. 12. f(a) = g(b)
. ⊢   by Witness a
. |
. ⊢  (f1 o f)(a) = b  by RWH rem_composeC 0
. |
. ⊢  f1(f(a)) = b  by RWH (HypC 12) 0
. |
. ⊢  f1(g(b)) = b  by RWH add_composeC 0
. |
. ⊢  (f1 o g)(b) = b  by RWH (HypC 9) 0
. |
. ⊢  Id(b) = b  by Reduce 0
END
```