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discrete__Formula__with_rank  THM

⊢  Discrete{Formula}  by Analyze 0
|
1. x : Formula
⊢  ∀y:Formula. Dec(x = y)  by FormulaRankInd -1
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| 2. x2 : Var
| ⊢  ∀y:Formula. Dec(⌈x2⌉ = y)  by Analyze 0
| |
| 3. y : Formula
| ⊢  Dec(⌈x2⌉ = y)
| | by let Solver ≡ FalseCase THEN FormulaEquality -1 in
| |    FormulaCases -1 THENL [Id;Solver;Solver;Solver;Solver]
| |
| 4. x3 : Var
| ⊢  Dec(⌈x2⌉ = ⌈x3⌉)  by Decide (x2 = x3)
| |
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| | 5. x2 = x3
| | ⊢  Dec(⌈x2⌉ = ⌈x3⌉)
| |   by TrueCase THEN FormulaEquality 0 THEN Trivial
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| . 5. ¬x2 = x3
| . ⊢  Dec(⌈x2⌉ = ⌈x3⌉)  by FalseCase THEN Analyze -2
| . |
| . 5. ⌈x2⌉ = ⌈x3⌉
| . ⊢  x2 = x3  by FormulaEquality -1
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| 2. x1 : Formula
| 3. ∀y:Formula. Dec(x1 = y)
| ⊢  ∀y@0:Formula. Dec(⌈◇⌉x1 = y@0)  by Analyze 0
| |
| 4. y@0 : Formula
| ⊢  Dec(⌈◇⌉x1 = y@0)
| | by let Solver ≡ FalseCase THEN FormulaEquality -1 in
| |    FormulaCases -1 THENL [Solver;Id;Solver;Solver;Solver]
| |
| 5. x2 : Formula
| ⊢  Dec(⌈◇⌉x1 = ⌈◇⌉x2)  by With x2 (Analyze -3)
| |
| 3. Formula
| 4. x2 : Formula
| 5. Dec(x1 = x2)
| ⊢  Dec(⌈◇⌉x1 = ⌈◇⌉x2)  by Analyze -1
| |
| |\
| | 5. x1 = x2
| | ⊢  Dec(⌈◇⌉x1 = ⌈◇⌉x2)
| |   by TrueCase THEN FormulaEquality 0 THEN Trivial
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| . 5. ¬x1 = x2
| . ⊢  Dec(⌈◇⌉x1 = ⌈◇⌉x2)  by FalseCase THEN Analyze -2
| . |
| . 5. ⌈◇⌉x1 = ⌈◇⌉x2
| . ⊢  x1 = x2  by FormulaEquality -1 THEN Trivial
|
|\
| 2. x2 : Formula
| 3. x3 : Formula
| 4. ∀y:Formula. Dec(x2 = y)
| 5. ∀y:Formula. Dec(x3 = y)
| ⊢  ∀y@0:Formula. Dec(x2⌈∧⌉x3 = y@0)  by Analyze 0
| |
| 6. y@0 : Formula
| ⊢  Dec(x2⌈∧⌉x3 = y@0)
| | by let Solver ≡ FalseCase THEN FormulaEquality -1 in
| |    FormulaCases -1 THENL [Solver;Solver;Id;Solver;Solver]
| |
| 7. x5 : Formula
| 8. x6 : Formula
| ⊢  Dec(x2⌈∧⌉x3 = x5⌈∧⌉x6)  by DWith -2 -5 THEN DWith -2 -5
| |
| 4. Formula
| 5. x5 : Formula
| 6. x6 : Formula
| 7. Dec(x2 = x5)
| 8. Dec(x3 = x6)
| ⊢  Dec(x2⌈∧⌉x3 = x5⌈∧⌉x6)  by Analyze -2
| |
| |\
| | 7. x2 = x5
| | 8. Dec(x3 = x6)
| | ⊢  Dec(x2⌈∧⌉x3 = x5⌈∧⌉x6)  by Analyze -1
| | |
| | |\
| | | 8. x3 = x6
| | | ⊢  Dec(x2⌈∧⌉x3 = x5⌈∧⌉x6)
| | |   by TrueCase THEN FormulaEquality 0 THEN Trivial
| | |
| | .\
| | . 8. ¬x3 = x6
| | . ⊢  Dec(x2⌈∧⌉x3 = x5⌈∧⌉x6)  by FalseCase THEN Analyze -2
| | . |
| | . 8. x2⌈∧⌉x3 = x5⌈∧⌉x6
| | . ⊢  x3 = x6  by FormulaEquality -1
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| . 7. ¬x2 = x5
| . 8. Dec(x3 = x6)
| . ⊢  Dec(x2⌈∧⌉x3 = x5⌈∧⌉x6)  by FalseCase THEN Analyze -3
| . |
| . 7. Dec(x3 = x6)
| . 8. x2⌈∧⌉x3 = x5⌈∧⌉x6
| . ⊢  x2 = x5  by FormulaEquality -1 THEN Trivial
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| 2. x2 : Formula
| 3. x3 : Formula
| 4. ∀y:Formula. Dec(x2 = y)
| 5. ∀y:Formula. Dec(x3 = y)
| ⊢  ∀y@0:Formula. Dec(x2⌈∨⌉x3 = y@0)  by Analyze 0
| |
| 6. y@0 : Formula
| ⊢  Dec(x2⌈∨⌉x3 = y@0)
| | by let Solver ≡ FalseCase THEN FormulaEquality -1 in
| |    FormulaCases -1 THENL [Solver;Solver;Solver;Id;Solver]
| |
| 7. x5 : Formula
| 8. x6 : Formula
| ⊢  Dec(x2⌈∨⌉x3 = x5⌈∨⌉x6)  by DWith -2 -5 THEN DWith -2 -5
| |
| 4. Formula
| 5. x5 : Formula
| 6. x6 : Formula
| 7. Dec(x2 = x5)
| 8. Dec(x3 = x6)
| ⊢  Dec(x2⌈∨⌉x3 = x5⌈∨⌉x6)  by Analyze -2
| |
| |\
| | 7. x2 = x5
| | 8. Dec(x3 = x6)
| | ⊢  Dec(x2⌈∨⌉x3 = x5⌈∨⌉x6)  by Analyze -1
| | |
| | |\
| | | 8. x3 = x6
| | | ⊢  Dec(x2⌈∨⌉x3 = x5⌈∨⌉x6)
| | |   by TrueCase THEN FormulaEquality 0 THEN Trivial
| | |
| | .\
| | . 8. ¬x3 = x6
| | . ⊢  Dec(x2⌈∨⌉x3 = x5⌈∨⌉x6)  by FalseCase THEN Analyze -2
| | . |
| | . 8. x2⌈∨⌉x3 = x5⌈∨⌉x6
| | . ⊢  x3 = x6  by FormulaEquality -1
| |
| .\
| . 7. ¬x2 = x5
| . 8. Dec(x3 = x6)
| . ⊢  Dec(x2⌈∨⌉x3 = x5⌈∨⌉x6)  by FalseCase THEN Analyze -3
| . |
| . 7. Dec(x3 = x6)
| . 8. x2⌈∨⌉x3 = x5⌈∨⌉x6
| . ⊢  x2 = x5  by FormulaEquality -1 THEN Trivial
|
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. 2. y1 : Formula
. 3. y2 : Formula
. 4. ∀y:Formula. Dec(y1 = y)
. 5. ∀y:Formula. Dec(y2 = y)
. ⊢  ∀y@0:Formula. Dec(y1⌈⇒⌉y2 = y@0)  by Analyze 0
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. 6. y@0 : Formula
. ⊢  Dec(y1⌈⇒⌉y2 = y@0)
. | by let Solver ≡ FalseCase THEN FormulaEquality -1 in
. |    FormulaCases -1 THENL [Solver;Solver;Solver;Solver;Id]
. |
. 7. y3 : Formula
. 8. y4 : Formula
. ⊢  Dec(y1⌈⇒⌉y2 = y3⌈⇒⌉y4)  by DWith -2 -5 THEN DWith -2 -5
. |
. 4. Formula
. 5. y3 : Formula
. 6. y4 : Formula
. 7. Dec(y1 = y3)
. 8. Dec(y2 = y4)
. ⊢  Dec(y1⌈⇒⌉y2 = y3⌈⇒⌉y4)  by Analyze -2
. |
. |\
. | 7. y1 = y3
. | 8. Dec(y2 = y4)
. | ⊢  Dec(y1⌈⇒⌉y2 = y3⌈⇒⌉y4)  by Analyze -1
. | |
. | |\
. | | 8. y2 = y4
. | | ⊢  Dec(y1⌈⇒⌉y2 = y3⌈⇒⌉y4)
. | |   by TrueCase THEN FormulaEquality 0 THEN Trivial
. | |
. | .\
. | . 8. ¬y2 = y4
. | . ⊢  Dec(y1⌈⇒⌉y2 = y3⌈⇒⌉y4)  by FalseCase THEN Analyze -2
. | . |
. | . 8. y1⌈⇒⌉y2 = y3⌈⇒⌉y4
. | . ⊢  y2 = y4  by FormulaEquality -1
. |
. .\
. . 7. ¬y1 = y3
. . 8. Dec(y2 = y4)

. . ⊢  Dec(y1⌈⇒⌉y2 = y3⌈⇒⌉y4)  by FalseCase THEN Analyze -3
. . |
. . 7. Dec(y2 = y4)
. . 8. y1⌈⇒⌉y2 = y3⌈⇒⌉y4
. . ⊢  y1 = y3  by FormulaEquality -1 THEN Trivial
END
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