```
absval_mul  THM

⊢  ∀a,b:ℤ. |a·b| = |a|·|b|  by Auto THEN (CaseNat 0 ‘a’)
|
|\
| 1. a : ℤ
| 2. b : ℤ
| 3. a = 0
| ⊢  |0·b| = |0|·|b|
|   by (Subst (0·b = 0) 0) THEN (Subst (|0| = 0) 0)
|      THEN
|      (Unfold ‘absval‘ 0)
|      THEN
|      (Reduce 0)
|
.\
. 1. a : ℤ
. 2. b : ℤ
. 3. ¬a = 0
. ⊢  |a·b| = |a|·|b|  by CaseNat 0 ‘b’
. |
. |\
. | 1. a : ℤ
. | 2. b : ℤ
. | 3. ¬a = 0
. | 4. b = 0
. | ⊢  |a·0| = |a|·|0|
. |   by (Subst (a·0 = 0) 0) THEN (Subst (|0| = 0) 0)
. |      THEN
. |      (Unfold ‘absval‘ 0)
. |      THEN
. |      (Reduce 0)
. |
. .\
. . 1. a : ℤ
. . 2. b : ℤ
. . 3. ¬a = 0
. . 4. ¬b = 0
. . ⊢  |a·b| = |a|·|b|
. . | by (Unfold ‘absval‘ 0) THEN (Repeat SplitOnConclITE)
. . |
. . |\
. . | .....falsecase .....
. . | 1. a : ℤ
. . | 2. b : ℤ
. . | 3. ¬a = 0
. . | 4. ¬b = 0
. . | 5. 0≤a·b
. . | 6. 0≤a
. . | 7. b<0
. . | ⊢  a·b = a·(-b)
. . |   by (AssertBY (a·b<0)
. . |       (BackThru
. . |        Thm* ∀a,b:ℤ. a·b<0 ⟺ a<0 & b>0 ∨ a>0 & b<0
. . |        {neg_mul_arg_bounds}))
```

```
. . |      THEN
. . |      (Try (OrLeft THEN (Complete Auto)))
. . |      THEN
. . |      (Try (OrRight THEN (Complete Auto)))
. . |
. . |\
. . | .....truecase .....
. . | 1. a : ℤ
. . | 2. b : ℤ
. . | 3. ¬a = 0
. . | 4. ¬b = 0
. . | 5. 0≤a·b
. . | 6. a<0
. . | 7. 0≤b
. . | ⊢  a·b = (-a)·b
. . |   by (AssertBY (a·b<0)
. . |       (BackThru
. . |        Thm* ∀a,b:ℤ. a·b<0 ⟺ a<0 & b>0 ∨ a>0 & b<0
. . |        {neg_mul_arg_bounds}))
. . |      THEN
. . |      (Try (OrLeft THEN (Complete Auto)))
. . |      THEN
. . |      (Try (OrRight THEN (Complete Auto)))
. . |
. . |\
. . | .....truecase .....
. . | 1. a : ℤ
. . | 2. b : ℤ
. . | 3. ¬a = 0
. . | 4. ¬b = 0
. . | 5. a·b<0
. . | 6. 0≤a
. . | 7. 0≤b
. . | ⊢  -a·b = a·b
. . |   by (AssertBY (a·b>0)
. . |       (BackThru
. . |        Thm* ∀a,b:ℤ. a·b>0 ⟺ a>0 & b>0 ∨ a<0 & b<0
. . |        {pos_mul_arg_bounds}))
. . |      THEN
. . |      (Try (OrLeft THEN (Complete Auto)))
. . |      THEN
. . |      (Try (OrRight THEN (Complete Auto)))
. . |
. . .\
. . . .....falsecase .....
. . . 1. a : ℤ
. . . 2. b : ℤ
. . . 3. ¬a = 0
. . . 4. ¬b = 0
. . . 5. a·b<0
. . . 6. a<0
. . . 7. b<0
. . . ⊢  -a·b = (-a)·(-b)
. . .   by (AssertBY (a·b>0)
. . .       (BackThru
```

```
. . .         Thm* ∀a,b:ℤ. a·b>0 ⟺ a>0 & b>0 ∨ a<0 & b<0
. . .         {pos_mul_arg_bounds}))
. . .       THEN
. . .       (Try (OrLeft THEN (Complete Auto)))
. . .       THEN
. . .       (Try (OrRight THEN (Complete Auto)))
END
```