```
div_rem_properties  THM

⊢  ∀a:ℤ, n:ℤ⁻⁰.
|  a = (a ÷ n)·n+(a rem n)
|  & |a rem n|<|n|
|  & ((a rem n)<0 ⇒ a<0)
|  & ((a rem n)>0 ⇒ a>0)
| by (RepeatFor 2 (Analyze 0))
|    THEN
|    (Inst Thm* ∀a:ℤ, n:ℤ⁻⁰. a = (a ÷ n)·n+(a rem n)    {div_rem_sum}
|     [a;n])
|    THEN
|    (Assert
|     (|a rem n|<|n| & ((a rem n)<0 ⇒ a<0) & ((a rem n)>0 ⇒ a>0)))
|
|\
| .....assertion .....
| 1. a : ℤ
| 2. n : ℤ⁻⁰
| 3. a = (a ÷ n)·n+(a rem n)
| ⊢  |a rem n|<|n| & ((a rem n)<0 ⇒ a<0) & ((a rem n)>0 ⇒ a>0)
| | by Assert ((0 ÷ n) = 0)
| |
| |\
| | .....assertion .....
| | 1. a : ℤ
| | 2. n : ℤ⁻⁰
| | 3. a = (a ÷ n)·n+(a rem n)
| | ⊢  (0 ÷ n) = 0  by Lemmaize []
| | |
| | 1. n : ℤ⁻⁰
| | ⊢  (0 ÷ n) = 0
| | | by ((Inst
| | |      Thm* ∀a:ℤ, n:ℤ⁻⁰. a = (a ÷ n)·n+(a rem n)    {div_rem_sum}
| | |      [0;n])
| | |     THEN
| | |     (Decide (0≤n)))
| | |    THENL
| | |    [Inst
| | |     Thm* ∀a:ℕ, n:ℕ⁺. 0≤(a rem n) & (a rem n)<n    {rem_bounds_1}
| | |     [0;n]
| | |    ;Inst
| | |     Thm* ∀a:{...0}, n:{...-1}. 0≥(a rem n) & (a rem n)>n
| | |     {rem_bounds_3}
| | |     [0;n]]
| | |
| | |\
| | | 1. n : ℤ⁻⁰
| | | 2. 0 = (0 ÷ n)·n+(0 rem n)
| | | 3. 0≤n
| | | 4. 0≤(0 rem n) & (0 rem n)<n
| | | ⊢  (0 ÷ n) = 0
| | |   by (Decide (1≤(0 ÷ n))) THEN (Decide ((0 ÷ n)≤-1))
| | |      THEN
| | |      (AllHyps
```

```
| | |       (λh.
| | |        Using [‘n’,n]
| | |        (FwdThru
| | |         Thm* ∀a,b:ℤ, n:ℕ. a≤b ⇒ n·a≤n·b    {mul_preserves_le}
| | |         [h])))
| | |
| | .\
| | . 1. n : ℤ⁻⁰
| | . 2. 0 = (0 ÷ n)·n+(0 rem n)
| | . 3. ¬0≤n
| | . 4. 0≥(0 rem n) & (0 rem n)>n
| | . ⊢  (0 ÷ n) = 0
| | .   by (Decide (1≤(0 ÷ n))) THEN (Decide ((0 ÷ n)≤-1))
| | .      THEN
| | .      (AllHyps
| | .       (λh.
| | .        Using [‘n’,(-n)]
| | .        (FwdThru
| | .         Thm* ∀a,b:ℤ, n:ℕ. a≤b ⇒ n·a≤n·b    {mul_preserves_le}
| | .         [h])))
| |
| .\
| . 1. a : ℤ
| . 2. n : ℤ⁻⁰
| . 3. a = (a ÷ n)·n+(a rem n)
| . 4. (0 ÷ n) = 0
| . ⊢  |a rem n|<|n| & ((a rem n)<0 ⇒ a<0) & ((a rem n)>0 ⇒ a>0)
| .   by (((Decide (0≤a)) THEN (Decide (0≤n)))
| .       THENL
| .       [Inst
| .        Thm* ∀a:ℕ, n:ℕ⁺. 0≤(a rem n) & (a rem n)<n
| .        {rem_bounds_1}
| .        [a;n]
| .       ;Inst
| .        Thm* ∀a:ℕ, n:{...-1}. 0≤(a rem n) & (a rem n)<-n
| .        {rem_bounds_4}
| .        [a;n]
| .       ;Inst
| .        Thm* ∀a:{...0}, n:ℕ⁺. 0≥(a rem n) & (a rem n)>-n
| .        {rem_bounds_2}
| .        [a;n]
| .       ;Inst
| .        Thm* ∀a:{...0}, n:{...-1}. 0≥(a rem n) & (a rem n)>n
| .        {rem_bounds_3}
| .        [a;n]])
| .      THEN
| .      (Unfold ‘absval‘ 0)
| .      THEN
| .      (Repeat SplitOnConclITE)
| .      THEN
| .      (MoveToConcl 3)
| .      THEN
| .      (MoveToConcl -1)
| .      THEN
| .      (CaseNat 0 ‘a’)
```

```
| .       THEN
| .       (Subst ((0 ÷ n) = 0) -1)
|
.\
. 1. a : ℤ
. 2. n : ℤ⁻⁰
. 3. a = (a ÷ n)·n+(a rem n)
. 4. |a rem n|<|n| & ((a rem n)<0 ⇒ a<0) & ((a rem n)>0 ⇒ a>0)
. ⊢  a = (a ÷ n)·n+(a rem n)
.    & |a rem n|<|n|
.    & ((a rem n)<0 ⇒ a<0)
.    & ((a rem n)>0 ⇒ a>0)  by Auto THEN ThinTrivial
END
```