f91-val THM

```
⊢  ∀i:ℤ. f91(i) ∼ if 101<_2i→ i-10 else 91 fi
| by (InductionOn (d = if 101<_2i→ 0 else 101-i fi) CompNatInd)
|    THENA
|    SplitOnConclITE
|
1. d : ℕ
2. ∀d1:ℕ.
|  d1<d
|  ⇒
|  (∀i:ℤ.
|   if 101<_2i→ 0 else 101-i fi = d1
|   ⇒
|   (f91(i) ∼ if 101<_2i→ i-10 else 91 fi))
⊢  ∀i:ℤ.
|  if 101<_2i→ 0 else 101-i fi = d
|  ⇒
|  (f91(i) ∼ if 101<_2i→ i-10 else 91 fi)
| by (Analyze 0) THEN SplitOnConclITE THEN (RecUnfold ‘f91‘ 0)
|    THEN
|    SplitOnConclITE
|    THEN
|    (Try (Complete Auto))
|
.....falsecase .....
1. d : ℕ
2. ∀d1:ℕ.
|  d1<d
|  ⇒
|  (∀i:ℤ.
|   if 101<_2i→ 0 else 101-i fi = d1
|   ⇒
|   (f91(i) ∼ if 101<_2i→ i-10 else 91 fi))
3. i : ℤ
4. i≤101
5. i≤100
⊢  101-i = d ⇒ (f91(f91(i+11)) ∼ 91)
| by (((Analyze 0)
|      THEN
|      (InstHyp [if 101<_2i+11→ 0 else 101-(i+11) fi;i+11] 2))
|     THENA
|     (Try (Complete Auto ORELSE Complete SplitOnConclITE)))
|    THEN
|    (HypSubst -1 0)
|
1. d : ℕ
2. ∀d1:ℕ.
|  d1<d
|  ⇒
|  (∀i:ℤ.
|   if 101<_2i→ 0 else 101-i fi = d1
|   ⇒
|   (f91(i) ∼ if 101<_2i→ i-10 else 91 fi))
3. i : ℤ
```

```
4. i≤101
5. i≤100
6. 101-i = d
7. f91(i+11) ~ if 101<_2i+11→ i+11-10 else 91 fi
⊢  f91(if 101<_2i+11→ i+11-10 else 91 fi) ~ 91  by SplitOnConclITE
|
|\
| .....truecase .....
| 1. d : ℕ
| 2. ∀d1:ℕ.
| |  d1<d
| |  ⇒
| |  (∀i:ℤ.
| |   if 101<_2i→ 0 else 101-i fi = d1
| |   ⇒
| |   (f91(i) ~ if 101<_2i→ i-10 else 91 fi))
| 3. i : ℤ
| 4. i≤101
| 5. i≤100
| 6. 101-i = d
| 7. f91(i+11) ~ if 101<_2i+11→ i+11-10 else 91 fi
| 8. 101<i+11
| ⊢  f91(i+11-10) ~ 91  by InstHyp [d-1;i+1] 2
| |
| |\
| | .....antecedent .....
| | 1. d : ℕ
| | 2. ∀d1:ℕ.
| |    d1<d
| |    ⇒
| |    (∀i:ℤ.
| |     if 101<_2i→ 0 else 101-i fi = d1
| |     ⇒
| |     (f91(i) ~ if 101<_2i→ i-10 else 91 fi))
| | 3. i : ℤ
| | 4. i≤101
| | 5. i≤100
| | 6. 101-i = d
| | 7. f91(i+11) ~ if 101<_2i+11→ i+11-10 else 91 fi
| | 8. 101<i+11
| | ⊢  if 101<_2i+1→ 0 else 101-(i+1) fi = d-1  by SplitOnConclITE
| |
| .\
| . 1. d : ℕ
| . 2. ∀d1:ℕ.
| . |  d1<d
| . |  ⇒
| . |  (∀i:ℤ.
| . |   if 101<_2i→ 0 else 101-i fi = d1
| . |   ⇒
| . |   (f91(i) ~ if 101<_2i→ i-10 else 91 fi))
| . 3. i : ℤ
| . 4. i≤101
| . 5. i≤100
| . 6. 101-i = d
```

```
| . 7. f91(i+11) ∼ if 101<_2i+11→ i+11-10 else 91 fi
| . 8. 101<i+11
| . 9. f91(i+1) ∼ if 101<_2i+1→ i+1-10 else 91 fi
| . ⊢  f91(i+11-10) ∼ 91  by (NthHypSq -1) THEN Analyze
| . |
| . |\
| . | 1. d : ℕ
| . | 2. ∀d1:ℕ.
| . |    d1<d
| . |    ⇒
| . |    (∀i:ℤ.
| . |     if 101<_2i→ 0 else 101-i fi = d1
| . |     ⇒
| . |     (f91(i) ∼ if 101<_2i→ i-10 else 91 fi))
| . | 3. i : ℤ
| . | 4. i≤101
| . | 5. i≤100
| . | 6. 101-i = d
| . | 7. f91(i+11) ∼ if 101<_2i+11→ i+11-10 else 91 fi
| . | 8. 101<i+11
| . | 9. f91(i+1) ∼ if 101<_2i+1→ i+1-10 else 91 fi
| . | ⊢  f91(i+11-10) ∼ f91(i+1)  by Analyze
| . |
| . .\
| . . 1. d : ℕ
| . . 2. ∀d1:ℕ.
| . .    d1<d
| . .    ⇒
| . .    (∀i:ℤ.
| . .     if 101<_2i→ 0 else 101-i fi = d1
| . .     ⇒
| . .     (f91(i) ∼ if 101<_2i→ i-10 else 91 fi))
| . . 3. i : ℤ
| . . 4. i≤101
| . . 5. i≤100
| . . 6. 101-i = d
| . . 7. f91(i+11) ∼ if 101<_2i+11→ i+11-10 else 91 fi
| . . 8. 101<i+11
| . . 9. f91(i+1) ∼ if 101<_2i+1→ i+1-10 else 91 fi
| . . ⊢  91 ∼ if 101<_2i+1→ i+1-10 else 91 fi  by SplitOnConclITE
|
.\
. .....falsecase .....
. 1. d : ℕ
. 2. ∀d1:ℕ.
.    d1<d
.    ⇒
.    (∀i:ℤ.
.     if 101<_2i→ 0 else 101-i fi = d1
.     ⇒
.     (f91(i) ∼ if 101<_2i→ i-10 else 91 fi))
. 3. i : ℤ
. 4. i≤101
. 5. i≤100
. 6. 101-i = d
```

```
. 7. f91(i+11) ∼ if 101<₂i+11→ i+11-10 else 91 fi
. 8. i+11≤101
. ⊢  f91(91) ∼ 91
.   by ((InstHyp [10;91] 2) THENA (Reduce 0)) THEN (HypSubst -1 0)
.      THEN
.      SplitOnConclITE
END
```