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length_le  THM

⊢  ∀T:Type, A,B:T List.
|  no_repeats(T;A) ⇒ (∀x:T. (x ∈ A) ⇒ (x ∈ B)) ⇒ ||A||≤||B||
| by Auto
|    THEN
|    (BackThru
|     Thm* ∀k,m:ℕ. (∃f:(ℕk→ℕm). Inj(ℕk; ℕm; f)) ⇒ k≤m
|     {injection_le})
|
1. T : Type
2. A : T List
3. B : T List
4. no_repeats(T;A)
5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
⊢  ∃f:(ℕ||A||→ℕ||B||). Inj(ℕ||A||; ℕ||B||; f)
| by Assert (∀i:ℕ||A||. ∃j:ℕ||B||. A[i] = B[j])
|
|\
| .....assertion .....
| 1. T : Type
| 2. A : T List
| 3. B : T List
| 4. no_repeats(T;A)
| 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
| ⊢  ∀i:ℕ||A||. ∃j:ℕ||B||. A[i] = B[j]
| | by (Analyze 0) THEN (InstHyp [A[i]] -2)
| |
| |\
| | .....antecedent .....
| | 1. T : Type
| | 2. A : T List
| | 3. B : T List
| | 4. no_repeats(T;A)
| | 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
| | 6. i : ℕ||A||
| | ⊢  (A[i] ∈ A)  by Obvious
| |
| .\
| . 1. T : Type
| . 2. A : T List
| . 3. B : T List
| . 4. no_repeats(T;A)
| . 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
| . 6. i : ℕ||A||
| . 7. (A[i] ∈ B)
| . ⊢  ∃j:ℕ||B||. A[i] = B[j]  by (Analyze -1) THEN Obvious
|
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. 1. T : Type
. 2. A : T List
. 3. B : T List
. 4. no_repeats(T;A)
. 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
. 6. ∀i:ℕ||A||. ∃j:ℕ||B||. A[i] = B[j]
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. ⊢  ∃f:(ℕ||A||→ℕ||B||). Inj(ℕ||A||; ℕ||B||; f)
. | by (RenameVar ‘f’ -1) THEN (Unfolds [‘all‘;‘exists‘] -1)
. |    THEN
. |    (InstConcl [λi.1of(f(i))])
. |    THEN
. |    (Unfold ‘inject‘ 0)
. |    THEN
. |    (Reduce 0)
. |    THEN
. |    (RepeatFor 2 (Analyze 0))
. |
. 1. T : Type
. 2. A : T List
. 3. B : T List
. 4. no_repeats(T;A)
. 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
. 6. f : i:ℕ||A||→j:ℕ||B||×A[i] = B[j]
. 7. a1 : ℕ||A||
. 8. a2 : ℕ||A||
. ⊢  1of(f(a1)) = 1of(f(a2)) ⇒ a1 = a2
. | by (GenConclAtAddr [1;2;1]) THEN (Thin -1) THEN (Analyze -1)
. |    THEN
. |    (Reduce 0)
. |    THEN
. |    (GenConclAtAddr [1;3;1])
. |    THEN
. |    (Thin -1)
. |    THEN
. |    (Analyze -1)
. |    THEN
. |    (Reduce 0)
. |
. 1. T : Type
. 2. A : T List
. 3. B : T List
. 4. no_repeats(T;A)
. 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
. 6. i:ℕ||A||→j:ℕ||B||×A[i] = B[j]
. 7. a1 : ℕ||A||
. 8. a2 : ℕ||A||
. 9. j : ℕ||B||
. 10. A[a1] = B[j]
. 11. j1 : ℕ||B||
. 12. A[a2] = B[j1]
. ⊢  j = j1 ⇒ a1 = a2  by Auto THEN (RevHypSubst -1 -2)
. |
. 1. T : Type
. 2. A : T List
. 3. B : T List
. 4. no_repeats(T;A)
. 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
. 6. i:ℕ||A||→j:ℕ||B||×A[i] = B[j]
. 7. a1 : ℕ||A||
. 8. a2 : ℕ||A||
. 9. j : ℕ||B||
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. 10. A[a1] = B[j]
. 11. j1 : ℕ||B||
. 12. A[a2] = B[j]
. 13. j = j1
. ⊢  a1 = a2
. | by SupposeNot THEN (Unfold ‘no_repeats‘ 4)
. |    THEN
. |    (InstHyp [a1;a2] 4)
. |
. 1. T : Type
. 2. A : T List
. 3. B : T List
. 4. ∀i,j:ℕ. i<||A|| ⇒ j<||A|| ⇒ ¬i = j ⇒ ¬A[i] = A[j]
. 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
. 6. i:ℕ||A||→j:ℕ||B||×A[i] = B[j]
. 7. a1 : ℕ||A||
. 8. a2 : ℕ||A||
. 9. j : ℕ||B||
. 10. A[a1] = B[j]
. 11. j1 : ℕ||B||
. 12. A[a2] = B[j]
. 13. j = j1
. 14. ¬a1 = a2
. 15. ¬A[a1] = A[a2]
. ⊢  a1 = a2  by Analyze -1
END
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