```
length_less  THM

⊢  ∀T:Type, A,B:T List.
|  no_repeats(T;A)
|  ⇒
|  (∀x:T. (x ∈ A) ⇒ (x ∈ B))
|  ⇒
|  (∃x:T. ¬(x ∈ A) & (x ∈ B)) ⇒ ||A||<||B||
| by Auto THEN ExRepD THEN (Analyze -1) THEN ExRepD
|    THEN
|    (Assert (∀k:ℕ||A||. ∃j:ℕ||B||. A[k] = B[j] & ¬j = i))
|
|\
| .....assertion .....
| 1. T : Type
| 2. A : T List
| 3. B : T List
| 4. no_repeats(T;A)
| 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
| 6. x : T
| 7. ¬(x ∈ A)
| 8. i : ℕ
| 9. i<||B||
| 10. x = B[i]
| ⊢  ∀k:ℕ||A||. ∃j:ℕ||B||. A[k] = B[j] & ¬j = i
| | by (Analyze 0) THEN (InstHyp [A[k]] 5)
| |
| |\
| | .....antecedent .....
| | 1. T : Type
| | 2. A : T List
| | 3. B : T List
| | 4. no_repeats(T;A)
| | 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
| | 6. x : T
| | 7. ¬(x ∈ A)
| | 8. i : ℕ
| | 9. i<||B||
| | 10. x = B[i]
| | 11. k : ℕ||A||
| | ⊢  (A[k] ∈ A)  by Obvious
| |
| .\
| . 1. T : Type
| . 2. A : T List
| . 3. B : T List
| . 4. no_repeats(T;A)
| . 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
| . 6. x : T
| . 7. ¬(x ∈ A)
| . 8. i : ℕ
| . 9. i<||B||
| . 10. x = B[i]
| . 11. k : ℕ||A||
| . 12. (A[k] ∈ B)
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| . ⊢  ∃j:ℕ||B||. A[k] = B[j] & ¬j = i
| . | by (Analyze -1) THEN ExRepD THEN (InstConcl [i1])
| . |
| . 1. T : Type
| . 2. A : T List
| . 3. B : T List
| . 4. no_repeats(T;A)
| . 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
| . 6. x : T
| . 7. ¬(x ∈ A)
| . 8. i : ℕ
| . 9. i<||B||
| . 10. x = B[i]
| . 11. k : ℕ||A||
| . 12. i1 : ℕ
| . 13. i1<||B||
| . 14. A[k] = B[i1]
| . ⊢  ¬i1 = i
| . | by (ParallelOp 7) THEN (Unfold `l_member` 0)
| . |    THEN
| . |    (InstConcl [k])
| . |
| . 1. T : Type
| . 2. A : T List
| . 3. B : T List
| . 4. no_repeats(T;A)
| . 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
| . 6. x : T
| . 7. i : ℕ
| . 8. i<||B||
| . 9. x = B[i]
| . 10. k : ℕ||A||
| . 11. i1 : ℕ
| . 12. i1<||B||
| . 13. A[k] = B[i1]
| . 14. i1 = i
| . ⊢  x = A[k]  by Obvious
|
.\
. 1. T : Type
. 2. A : T List
. 3. B : T List
. 4. no_repeats(T;A)
. 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
. 6. x : T
. 7. ¬(x ∈ A)
. 8. i : ℕ
. 9. i<||B||
. 10. x = B[i]
. 11. ∀k:ℕ||A||. ∃j:ℕ||B||. A[k] = B[j] & ¬j = i
. ⊢  ||A||<||B||
. | by (Assert (||A||≤||B||-1))
. |    THEN
. |    (BackThru
. |     Thm* ∀k,m:ℕ. (∃f:(ℕk→ℕm). Inj(ℕk; ℕm; f)) ⇒ k≤m
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. |     {injection_le})
. |
. 1. T : Type
. 2. A : T List
. 3. B : T List
. 4. no_repeats(T;A)
. 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
. 6. x : T
. 7. ¬(x ∈ A)
. 8. i : ℕ
. 9. i<||B||
. 10. x = B[i]
. 11. ∀k:ℕ||A||. ∃j:ℕ||B||. A[k] = B[j] & ¬j = i
. ⊢  ∃f:(ℕ||A||→ℕ(||B||-1)). Inj(ℕ||A||; ℕ(||B||-1); f)
. | by (RenameVar ‘f’ -1) THEN (Unfolds [‘all‘;‘exists‘] -1)
. |    THEN
. |    (InstConcl [λk.if 1of(f(k))<₂i→ 1of(f(k)) else 1of(f(k))-1 fi])
. |    THEN
. |    (Try (Complete Auto))
. |
. |\
. | .....wf .....
. | 1. T : Type
. | 2. A : T List
. | 3. B : T List
. | 4. no_repeats(T;A)
. | 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
. | 6. x : T
. | 7. ¬(x ∈ A)
. | 8. i : ℕ
. | 9. i<||B||
. | 10. x = B[i]
. | 11. f : k:ℕ||A||→j:ℕ||B||×A[k] = B[j] & ¬j = i
. | ⊢  (λk.if 1of(f(k))<₂i→ 1of(f(k)) else 1of(f(k))-1 fi)
. | |  ∈ ℕ||A||→ℕ(||B||-1)  by Analyze
. | |
. | .....subterm .....
. | 1. T : Type
. | 2. A : T List
. | 3. B : T List
. | 4. no_repeats(T;A)
. | 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
. | 6. x : T
. | 7. ¬(x ∈ A)
. | 8. i : ℕ
. | 9. i<||B||
. | 10. x = B[i]
. | 11. f : k:ℕ||A||→j:ℕ||B||×A[k] = B[j] & ¬j = i
. | 12. k : ℕ||A||
. | ⊢  if 1of(f(k))<₂i→ 1of(f(k)) else 1of(f(k))-1 fi ∈ ℕ(||B||-1)
. | | by (GenConclAtAddr [2;1;1;1]) THEN (Analyze -2) THEN (Reduce 0)
. | |
. | 1. T : Type
. | 2. A : T List
. | 3. B : T List
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. | 4. no_repeats(T;A)
. | 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
. | 6. x : T
. | 7. ¬(x ∈ A)
. | 8. i : ℕ
. | 9. i<||B||
. | 10. x = B[i]
. | 11. f : k:ℕ||A||→j:ℕ||B||×A[k] = B[j] & ¬j = i
. | 12. k : ℕ||A||
. | 13. j : ℕ||B||
. | 14. z1 : A[k] = B[j] & ¬j = i
. | 15. f(k) = <j,z1>
. | ⊢  if j<_2i→ j else j-1 fi ∈ ℕ(||B||-1)  by SplitOnConclITE
. |
. .\
. . 1. T : Type
. . 2. A : T List
. . 3. B : T List
. . 4. no_repeats(T;A)
. . 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
. . 6. x : T
. . 7. ¬(x ∈ A)
. . 8. i : ℕ
. . 9. i<||B||
. . 10. x = B[i]
. . 11. f : k:ℕ||A||→j:ℕ||B||×A[k] = B[j] & ¬j = i
. . ⊢  Inj(ℕ||A||; ℕ(||B||-1);
. . |       λk.if 1of(f(k))<_2i→ 1of(f(k)) else 1of(f(k))-1 fi)
. . | by (Unfold ‘inject‘ 0) THEN (Reduce 0)
. . |    THEN
. . |    (RepeatFor 2 (Analyze 0))
. . |    THEN
. . |    (GenConclAtAddr [1;2;1;1;1])
. . |    THEN
. . |    (Thin -1)
. . |    THEN
. . |    (Analyze -1)
. . |    THEN
. . |    (Reduce 0)
. . |    THEN
. . |    (GenConclAtAddr [1;3;1;1;1])
. . |    THEN
. . |    (Thin -1)
. . |    THEN
. . |    (Analyze -1)
. . |    THEN
. . |    (Reduce 0)
. . |
. . 1. T : Type
. . 2. A : T List
. . 3. B : T List
. . 4. no_repeats(T;A)
. . 5. ∀x:T. (x ∈ A) ⇒ (x ∈ B)
. . 6. x : T
. . 7. ¬(x ∈ A)
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. . 8. i : ℕ
. . 9. i<||B||
. . 10. x = B[i]
. . 11. k:ℕ||A||→j:ℕ||B||×A[k] = B[j] & ¬j = i
. . 12. a1 : ℕ||A||
. . 13. a2 : ℕ||A||
. . 14. j : ℕ||B||
. . 15. A[a1] = B[j] & ¬j = i
. . 16. j1 : ℕ||B||
. . 17. A[a2] = B[j1] & ¬j1 = i
. . ⊢  if j<₂i→ j else j-1 fi = if j1<₂i→ j1 else j1-1 fi ⇒ a1 = a2
. .   by (RepeatFor 2 SplitOnConclITE) THEN SupposeNot
. .      THEN
. .      (Unfold ‘no_repeats‘ 4)
. .      THEN
. .      (InstHyp [a1;a2] 4)
. .      THEN
. .      (Analyze -1)
. .      THEN
. .      (SubstFor A[a1] 0)
. .      THEN
. .      (SubstFor A[a2] 0)
. .      THEN
. .      Analyze
END
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