```
length_upto  THM

⊢  ∀j:ℕ, i:ℕj. ||upto(i;j)|| = j-i
| by Assert (∀d,j,i:ℕ. j-i = d ⇒ ||upto(i;j)|| = d)
|
|\
| .....assertion .....
| ⊢  ∀d,j,i:ℕ. j-i = d ⇒ ||upto(i;j)|| = d
| | by InductionOnNat THEN (RecUnfold `upto` 0) THEN (Reduce 0)
| |    THEN
| |    SplitOnConclITE
| |    THEN
| |    (Reduce 0)
| |
| 1. d : ℤ
| 2. 0<d
| 3. ∀j,i:ℕ. j-i = d-1 ⇒ ||upto(i;j)|| = d-1
| 4. j : ℕ
| 5. i : ℕ
| 6. j-i = d
| 7. i<j
| ⊢  ||upto(i+1;j)||+1 = d  by InstHyp [j;i+1] 3
|
.\
. 1. ∀d,j,i:ℕ. j-i = d ⇒ ||upto(i;j)|| = d
. ⊢  ∀j:ℕ, i:ℕj. ||upto(i;j)|| = j-i
.   by Auto THEN (InstHyp [j-i;j;i] 1)
END
```