```
list-decomp-last  THM

⊢  ∀T:Type, s:T List.
|  0<||s|| ⇒ (s ∼ (firstn(||s||-1;s) @ [s[(||s||-1)]]))
| by InductionOnList THEN (Reduce 0) THEN (Analyze 0)
|
|\
| 1. T : Type
| 2. T List
| 3. 0<0
| ⊢  nil ∼ (firstn(-1;nil) @ [nil[(-1)]])  by Auto
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.\
. 1. T : Type
. 2. T List
. 3. u : T
. 4. v : T List
. 5. 0<||v|| ⇒ (v ∼ (firstn(||v||-1;v) @ [v[(||v||-1)]]))
. 6. 0<||v||+1
. ⊢  [u / v] ∼ (firstn(||v||+1-1;[u / v]) @ [[u / v][(||v||+1-1)]])
. | by (Analyze -3) THEN (Reduce 0) THEN (RecUnfold ‘firstn‘ 0)
. |    THEN
. |    (Reduce 0)
. |    THEN
. |    (Try SplitOnConclITE)
. |    THEN
. |    (Reduce 0)
. |    THEN
. |    Analyze
. |    THEN
. |    (Try Trivial)
. |    THEN
. |    (Try (Complete Auto))
. |
. 1. T : Type
. 2. T List
. 3. u : T
. 4. u1 : T
. 5. v1 : T List
. 6. 0<||[u1 / v1]||
. |  ⇒
. |  ([u1 / v1] ∼ (firstn(||[u1 / v1]||-1;[u1 / v1])
. |   @ [[u1 / v1][(||[u1 / v1]||-1)]]))
. 7. 0<||[u1 / v1]||+1
. 8. 0<||v1||+1+1-1
. ⊢  [u1 / v1] ∼ (firstn(||v1||+1+1-1-1;[u1 / v1])
. |  @ [[u; u1 / v1][(||v1||+1+1-1)]])  by Analyze -3
. |
. 1. T : Type
. 2. T List
. 3. u : T
. 4. u1 : T
. 5. v1 : T List
. 6. 0<||[u1 / v1]||+1
. 7. 0<||v1||+1+1-1
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. 8. [u1 / v1] ∼ (firstn(||[u1 / v1]||-1;[u1 / v1])
. |  @ [[u1 / v1][(||[u1 / v1]||-1)]])
. ⊢  [u1 / v1] ∼ (firstn(||v1||+1+1-1-1;[u1 / v1])
. |  @ [[u; u1 / v1][(||v1||+1+1-1)]])
. | by (NthHypSq -1) THEN Analyze THEN Analyze
. |
. |\
. | 1. T : Type
. | 2. T List
. | 3. T
. | 4. u1 : T
. | 5. v1 : T List
. | 6. 0<||[u1 / v1]||+1
. | 7. 0<||v1||+1+1-1
. | 8. [u1 / v1] ∼ (firstn(||[u1 / v1]||-1;[u1 / v1])
. |    @ [[u1 / v1][(||[u1 / v1]||-1)]])
. | ⊢  firstn(||v1||+1+1-1-1;[u1 / v1]) ∼ firstn(||[u1 / v1]||
. |    -1;[u1 / v1])  by Analyze THEN (Reduce 0)
. |
. .\
. . 1. T : Type
. . 2. T List
. . 3. u : T
. . 4. u1 : T
. . 5. v1 : T List
. . 6. 0<||[u1 / v1]||+1
. . 7. 0<||v1||+1+1-1
. . 8. [u1 / v1] ∼ (firstn(||[u1 / v1]||-1;[u1 / v1])
. . |  @ [[u1 / v1][(||[u1 / v1]||-1)]])
. . ⊢  [[u; u1 / v1][(||v1||+1+1-1)]] ∼ [[u1 / v1][(||[u1 / v1]||-1)]]
. . | by Subst ((||v1||+1+1-1) ∼ (||v1||+1)) 0
. . |
. . |\
. . | .....equality .....
. . | 1. T : Type
. . | 2. T List
. . | 3. T
. . | 4. u1 : T
. . | 5. v1 : T List
. . | 6. 0<||[u1 / v1]||+1
. . | 7. 0<||v1||+1+1-1
. . | 8. [u1 / v1] ∼ (firstn(||[u1 / v1]||-1;[u1 / v1])
. . |    @ [[u1 / v1][(||[u1 / v1]||-1)]])
. . | ⊢  (||v1||+1+1-1) ∼ (||v1||+1)  by Auto
. . |
. . .\
. . . 1. T : Type
. . . 2. T List
. . . 3. u : T
. . . 4. u1 : T
. . . 5. v1 : T List
. . . 6. 0<||[u1 / v1]||+1
. . . 7. 0<||v1||+1+1-1
. . . 8. [u1 / v1] ∼ (firstn(||[u1 / v1]||-1;[u1 / v1])
. . . |  @ [[u1 / v1][(||[u1 / v1]||-1)]])
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. . . ⊢  [[u; u1 / v1][(||v1||+1)]] ∼ [[u1 / v1][(||[u1 / v1]||-1)]]
. . . | by Subst ([u; u1 / v1][(||v1||+1)] ∼ [u1 / v1][||v1||]) 0
. . . |
. . . |\
. . . | .....equality .....
. . . | 1. T : Type
. . . | 2. T List
. . . | 3. u : T
. . . | 4. u1 : T
. . . | 5. v1 : T List
. . . | 6. 0<||[u1 / v1]||+1
. . . | 7. 0<||v1||+1+1-1
. . . | 8. [u1 / v1] ∼ (firstn(||[u1 / v1]||-1;[u1 / v1])
. . . |    @ [[u1 / v1][(||[u1 / v1]||-1)]])
. . . | ⊢  [u; u1 / v1][(||v1||+1)] ∼ [u1 / v1][||v1||]  by Obvious
. . . |
. . . .\
. . . . 1. T : Type
. . . . 2. T List
. . . . 3. T
. . . . 4. u1 : T
. . . . 5. v1 : T List
. . . . 6. 0<||[u1 / v1]||+1
. . . . 7. 0<||v1||+1+1-1
. . . . 8. [u1 / v1] ∼ (firstn(||[u1 / v1]||-1;[u1 / v1])
. . . .    @ [[u1 / v1][(||[u1 / v1]||-1)]])
. . . . ⊢  [[u1 / v1][||v1||]] ∼ [[u1 / v1][(||[u1 / v1]||-1)]]
. . . .   by Analyze THEN Analyze THEN (Reduce 0)
END
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