```
mod_minus  THM

⊢  ∀x:ℤ, n:ℕ+ . ((-x) mod n) = if (x mod n)=₂0→ 0 else n-(x mod n) fi
| by Auto
|    THEN
|    (Inst
|     Thm* ∀a:ℤ, n:ℕ+ . a = (a ÷↓ n)·n+(a mod n) & (a mod n)<n
|     {div_floor_mod_properties}
|     [x;n])
|    THEN
|    SplitOnConclITE
|
|\
| .....truecase .....
| 1. x : ℤ
| 2. n : ℕ+
| 3. x = (x ÷↓ n)·n+(x mod n) & (x mod n)<n
| 4. (x mod n) = 0
| ⊢  ((-x) mod n) = 0
|   by Inst
|      Thm* ∀a:ℤ, n:ℕ+, q:ℤ, r:ℕ.
|           a = q·n+r ⇒ r<n ⇒ q = (a ÷↓ n) & r = (a mod n)
|      {div_floor_mod_unique}
|      [-x;n;-(x ÷↓ n);0]
|
.\
. .....falsecase .....
. 1. x : ℤ
. 2. n : ℕ+
. 3. x = (x ÷↓ n)·n+(x mod n) & (x mod n)<n
. 4. ¬(x mod n) = 0
. ⊢  ((-x) mod n) = n-(x mod n)
.   by Inst
.      Thm* ∀a:ℤ, n:ℕ+, q:ℤ, r:ℕ.
.           a = q·n+r ⇒ r<n ⇒ q = (a ÷↓ n) & r = (a mod n)
.      {div_floor_mod_unique}
.      [-x;n;-((x ÷↓ n)+1);n-(x mod n)]
END
```