```
no_repeats_append_iff  THM

⊢  ∀T:Type, l1,l2:T List.
|  no_repeats(T;l1 @ l2)
|  ⟺
|  l_disjoint(T;l1;l2) & no_repeats(T;l1) & no_repeats(T;l2)
| by Unfolds [‘no_repeats‘;‘l_disjoint‘] 0
|
|\
| 1. T : Type
| 2. l1 : T List
| 3. l2 : T List
| 4. ∀i,j:ℕ.
| |  i<||l1 @ l2||
| |  ⇒
| |  j<||l1 @ l2|| ⇒ ¬i = j ⇒ ¬(l1 @ l2)[i] = (l1 @ l2)[j]
| 5. x : T
| ⊢  ¬((x ∈ l1) & (x ∈ l2))
| | by (RWO
| |      Thm* ∀as,bs:T List. ||as @ bs|| = ||as||+||bs|| ∈ ℤ
| |      {length_append}
| |      4)
| |     THEN
| |     (Unfold ‘l_member‘ 0)
| |     THEN
| |     (Analyze 0)
| |     THEN
| |     ExRepD
| |     THEN
| |     (InstHyp [i1;i+||l1||] 4)
| |     THEN
| |     (Analyze -1)
| |
| 1. T : Type
| 2. l1 : T List
| 3. l2 : T List
| 4. ∀i,j:ℕ.
| |  i<||l1||+||l2||
| |  ⇒
| |  j<||l1||+||l2|| ⇒ ¬i = j ⇒ ¬(l1 @ l2)[i] = (l1 @ l2)[j]
| 5. x : T
| 6. i1 : ℕ
| 7. i1<||l1||
| 8. x = l1[i1]
| 9. i : ℕ
| 10. i<||l2||
| 11. x = l2[i]
| ⊢  (l1 @ l2)[i1] = (l1 @ l2)[(i+||l1||)]
| | by (RW
| |      (AddrC [2]
| |       (LemmaC
| |        Thm* ∀as,bs:T List, i:ℕ||as||. (as @ bs)[i] = as[i]
| |        {select_append_front}))
| |      0)
| |     THEN
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| |    (RWO
| |     Thm* ∀as,bs:T List, i:{||as||..(||as||+||bs||)⁻}.
| |          (as @ bs)[i] = bs[(i-||as||)]
| |     {select_append_back}
| |     0)
| |
| 1. T : Type
| 2. l1 : T List
| 3. l2 : T List
| 4. ∀i,j:ℕ.
|    i<||l1||+||l2||
|    ⇒
|    j<||l1||+||l2|| ⇒ ¬i = j ⇒ ¬(l1 @ l2)[i] = (l1 @ l2)[j]
| 5. x : T
| 6. i1 : ℕ
| 7. i1<||l1||
| 8. x = l1[i1]
| 9. i : ℕ
| 10. i<||l2||
| 11. x = l2[i]
| ⊢  l1[i1] = l2[(i+||l1||-||l1||)]  by Obvious
|
|\
| 1. T : Type
| 2. l1 : T List
| 3. l2 : T List
| 4. ∀i,j:ℕ.
|    i<||l1 @ l2||
|    ⇒
|    j<||l1 @ l2|| ⇒ ¬i = j ⇒ ¬(l1 @ l2)[i] = (l1 @ l2)[j]
| 5. i : ℕ
| 6. j : ℕ
| 7. i<||l1||
| 8. j<||l1||
| 9. ¬i = j
| ⊢  ¬l1[i] = l1[j]
|   by (RWO
|       Thm* ∀as,bs:T List. ||as @ bs|| = ||as||+||bs|| ∈ ℤ
|       {length_append}
|       4)
|      THEN
|      (InstHyp [i;j] 4)
|      THEN
|      (RWO
|       Thm* ∀as,bs:T List, i:ℕ||as||. (as @ bs)[i] = as[i]
|       {select_append_front}
|       -1)

|
|\
| 1. T : Type
| 2. l1 : T List
| 3. l2 : T List
| 4. ∀i,j:ℕ.
|    i<||l1 @ l2||
|    ⇒
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|    j<||l1 @ l2|| ⇒ ¬i = j ⇒ ¬(l1 @ l2)[i] = (l1 @ l2)[j]
| 5. i : ℕ
| 6. j : ℕ
| 7. i<||l2||
| 8. j<||l2||
| 9. ¬i = j
| ⊢  ¬l2[i] = l2[j]
|   by (RWO
|       Thm* ∀as,bs:T List. ||as @ bs|| = ||as||+||bs|| ∈ ℤ
|       {length_append}
|       4)
|      THEN
|      (InstHyp [i+||l1||;j+||l1||] 4)
|      THEN
|      (RWO
|       Thm* ∀as,bs:T List, i:{||as||..(||as||+||bs||)⁻}.
|            (as @ bs)[i] = bs[(i-||as||)]
|       {select_append_back}
|       -1)
|      THEN
|      (NthHypSq -1)
|      THEN
|      (Repeat Analyze)
|
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. 1. T : Type
. 2. l1 : T List
. 3. l2 : T List
. 4. ∀x:T. ¬((x ∈ l1) & (x ∈ l2))
. 5. ∀i,j:ℕ. i<||l1|| ⇒ j<||l1|| ⇒ ¬i = j ⇒ ¬l1[i] = l1[j]
. 6. ∀i,j:ℕ. i<||l2|| ⇒ j<||l2|| ⇒ ¬i = j ⇒ ¬l2[i] = l2[j]
. 7. i : ℕ
. 8. j : ℕ
. 9. i<||l1 @ l2||
. 10. j<||l1 @ l2||
. 11. ¬i = j
. ⊢  ¬(l1 @ l2)[i] = (l1 @ l2)[j]
. | by (((All
. |       (RWO
. |        Thm* ∀as,bs:T List. ||as @ bs|| = ||as||+||bs|| ∈ ℤ
. |        {length_append}))
. |      THEN
. |      (Decide (i<||l1||)))
. |     THENL
. |     [RW
. |      (AddrC [1;2]
. |       (LemmaC
. |        Thm* ∀as,bs:T List, i:ℕ||as||. (as @ bs)[i] = as[i]
. |        {select_append_front}))
. |      0
. |     ;RW
. |      (AddrC [1;2]
. |       (LemmaC
. |        Thm* ∀as,bs:T List, i:{||as||..(||as||+||bs||)⁻}.
. |             (as @ bs)[i] = bs[(i-||as||)]
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. |        {select_append_back}))
. |      0])
. |    THEN
. |    ((Decide (j<||l1||))
. |     THENL
. |     [RW
. |      (AddrC [1;3]
. |       (LemmaC
. |        Thm* ∀as,bs:T List, i:ℕ||as||. (as @ bs)[i] = as[i]
. |        {select_append_front}))
. |      0
. |     ;RW
. |      (AddrC [1;3]
. |       (LemmaC
. |        Thm* ∀as,bs:T List, i:{||as||..(||as||+||bs||)⁻}.
. |             (as @ bs)[i] = bs[(i-||as||)]
. |        {select_append_back}))
. |      0])
. |    THEN
. |    (Try (BackThruSomeHyp THEN (Complete Auto)))
. |
. |\
. | 1. T : Type
. | 2. l1 : T List
. | 3. l2 : T List
. | 4. ∀x:T. ¬((x ∈ l1) & (x ∈ l2))
. | 5. ∀i,j:ℕ. i<||l1|| ⇒ j<||l1|| ⇒ ¬i = j ⇒ ¬l1[i] = l1[j]
. | 6. ∀i,j:ℕ. i<||l2|| ⇒ j<||l2|| ⇒ ¬i = j ⇒ ¬l2[i] = l2[j]
. | 7. i : ℕ
. | 8. j : ℕ
. | 9. i<||l1||+||l2||
. | 10. j<||l1||+||l2||
. | 11. ¬i = j
. | 12. i<||l1||
. | 13. ¬j<||l1||
. | ⊢  ¬l1[i] = l2[(j-||l1||)]
. | | by (InstHyp [l1[i]] 4) THEN (ParallelOp -1)
. | |
. | |\
. | | 1. T : Type
. | | 2. l1 : T List
. | | 3. l2 : T List
. | | 4. ∀x:T. ¬((x ∈ l1) & (x ∈ l2))
. | | 5. ∀i,j:ℕ. i<||l1|| ⇒ j<||l1|| ⇒ ¬i = j ⇒ ¬l1[i] = l1[j]
. | | 6. ∀i,j:ℕ. i<||l2|| ⇒ j<||l2|| ⇒ ¬i = j ⇒ ¬l2[i] = l2[j]
. | | 7. i : ℕ
. | | 8. j : ℕ
. | | 9. i<||l1||+||l2||
. | | 10. j<||l1||+||l2||
. | | 11. ¬i = j
. | | 12. i<||l1||
. | | 13. ¬j<||l1||
. | | 14. l1[i] = l2[(j-||l1||)]
. | | ⊢  (l1[i] ∈ l1)  by Obvious
. | |
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. | .\
. | . 1. T : Type
. | . 2. l1 : T List
. | . 3. l2 : T List
. | . 4. ∀x:T. ¬((x ∈ l1) & (x ∈ l2))
. | . 5. ∀i,j:ℕ. i<||l1|| ⇒ j<||l1|| ⇒ ¬i = j ⇒ ¬l1[i] = l1[j]
. | . 6. ∀i,j:ℕ. i<||l2|| ⇒ j<||l2|| ⇒ ¬i = j ⇒ ¬l2[i] = l2[j]
. | . 7. i : ℕ
. | . 8. j : ℕ
. | . 9. i<||l1||+||l2||
. | . 10. j<||l1||+||l2||
. | . 11. ¬i = j
. | . 12. i<||l1||
. | . 13. ¬j<||l1||
. | . 14. l1[i] = l2[(j-||l1||)]
. | . ⊢  (l1[i] ∈ l2)  by (HypSubst -1 0) THEN Obvious
. |
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. . 1. T : Type
. . 2. l1 : T List
. . 3. l2 : T List
. . 4. ∀x:T. ¬((x ∈ l1) & (x ∈ l2))
. . 5. ∀i,j:ℕ. i<||l1|| ⇒ j<||l1|| ⇒ ¬i = j ⇒ ¬l1[i] = l1[j]
. . 6. ∀i,j:ℕ. i<||l2|| ⇒ j<||l2|| ⇒ ¬i = j ⇒ ¬l2[i] = l2[j]
. . 7. i : ℕ
. . 8. j : ℕ
. . 9. i<||l1||+||l2||
. . 10. j<||l1||+||l2||
. . 11. ¬i = j
. . 12. ¬i<||l1||
. . 13. j<||l1||
. . ⊢  ¬l2[(i-||l1||)] = l1[j]
. . | by (InstHyp [l1[j]] 4) THEN (ParallelOp -1)
. . |
. . |\
. . | 1. T : Type
. . | 2. l1 : T List
. . | 3. l2 : T List
. . | 4. ∀x:T. ¬((x ∈ l1) & (x ∈ l2))
. . | 5. ∀i,j:ℕ. i<||l1|| ⇒ j<||l1|| ⇒ ¬i = j ⇒ ¬l1[i] = l1[j]
. . | 6. ∀i,j:ℕ. i<||l2|| ⇒ j<||l2|| ⇒ ¬i = j ⇒ ¬l2[i] = l2[j]
. . | 7. i : ℕ
. . | 8. j : ℕ
. . | 9. i<||l1||+||l2||
. . | 10. j<||l1||+||l2||
. . | 11. ¬i = j
. . | 12. ¬i<||l1||
. . | 13. j<||l1||
. . | 14. l2[(i-||l1||)] = l1[j]
. . | ⊢  (l1[j] ∈ l1)  by Obvious
. . |
. . .\
. . . 1. T : Type
. . . 2. l1 : T List
. . . 3. l2 : T List
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. . . 4. ∀x:T. ¬((x ∈ l1) & (x ∈ l2))
. . . 5. ∀i,j:ℕ. i<||l1|| ⇒ j<||l1|| ⇒ ¬i = j ⇒ ¬l1[i] = l1[j]
. . . 6. ∀i,j:ℕ. i<||l2|| ⇒ j<||l2|| ⇒ ¬i = j ⇒ ¬l2[i] = l2[j]
. . . 7. i : ℕ
. . . 8. j : ℕ
. . . 9. i<||l1||+||l2||
. . . 10. j<||l1||+||l2||
. . . 11. ¬i = j
. . . 12. ¬i<||l1||
. . . 13. j<||l1||
. . . 14. l2[(i-||l1||)] = l1[j]
. . . ⊢  (l1[j] ∈ l2)  by (RevHypSubst -1 0) THEN Obvious
END
```