```
no_repeats_upto  THM

⊢  ∀i,j:ℕ. no_repeats(ℕ;upto(i;j))
| by Assert (∀d,i,j:ℕ. j-i = d ⇒ no_repeats(ℕ;upto(i;j)))
|
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| .....assertion .....
| ⊢  ∀d,i,j:ℕ. j-i = d ⇒ no_repeats(ℕ;upto(i;j))  by InductionOnNat
| |
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| | .....basecase .....
| | ⊢  ∀i,j:ℕ. j-i = 0 ⇒ no_repeats(ℕ;upto(i;j))
| | | by Auto THEN (RecUnfold `upto` 0) THEN SplitOnConclITE
| | |
| | .....falsecase .....
| | 1. i : ℕ
| | 2. j : ℕ
| | 3. j-i = 0
| | 4. j≤i
| | ⊢  no_repeats(ℕ;nil)  by ObviousConcl
| |
| .\
| . .....upcase .....
| . 1. d : ℤ
| . 2. 0<d
| . 3. ∀i,j:ℕ. j-i = d-1 ⇒ no_repeats(ℕ;upto(i;j))
| . ⊢  ∀i,j:ℕ. j-i = d ⇒ no_repeats(ℕ;upto(i;j))
| . | by Auto THEN (RecUnfold `upto` 0) THEN SplitOnConclITE
| . |
| . .....truecase .....
| . 1. d : ℤ
| . 2. 0<d
| . 3. ∀i,j:ℕ. j-i = d-1 ⇒ no_repeats(ℕ;upto(i;j))
| . 4. i : ℕ
| . 5. j : ℕ
| . 6. j-i = d
| . 7. i<j
| . ⊢  no_repeats(ℕ;[i / upto(i+1;j)])
| . | by (RWO
| . |     Thm* ∀l:T List, x:T.
| . |          no_repeats(T;[x / l]) ⟺ no_repeats(T;l) & ¬(x ∈ l)
| . |     {no_repeats_cons}
| . |     0)
| . |    THEN
| . |    EasyHyp
| . |
| . 1. d : ℤ
| . 2. 0<d
| . 3. ∀i,j:ℕ. j-i = d-1 ⇒ no_repeats(ℕ;upto(i;j))
| . 4. i : ℕ
| . 5. j : ℕ
| . 6. j-i = d
| . 7. i<j
| . ⊢  ¬(i ∈ upto(i+1;j))
| .   by (Analyze 0)
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| .      THEN
| .      (RWO
| .       Thm* ∀i,j,k:ℕ. (k ∈ upto(i;j)) ⟺ i≤k & k<j    {member_upto}
| .       -1)
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. 1. ∀d,i,j:ℕ. j-i = d ⇒ no_repeats(ℕ;upto(i;j))
. ⊢  ∀i,j:ℕ. no_repeats(ℕ;upto(i;j))
. | by (RepeatFor 2 (Analyze 0)) THEN (Decide (i≤j))
. |
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. | 1. ∀d,i,j:ℕ. j-i = d ⇒ no_repeats(ℕ;upto(i;j))
. | 2. i : ℕ
. | 3. j : ℕ
. | 4. i≤j
. | ⊢  no_repeats(ℕ;upto(i;j))  by InstHyp [j-i;i;j] 1
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. .\
. . 1. ∀d,i,j:ℕ. j-i = d ⇒ no_repeats(ℕ;upto(i;j))
. . 2. i : ℕ
. . 3. j : ℕ
. . 4. ¬i≤j
. . ⊢  no_repeats(ℕ;upto(i;j))
. . | by (RecUnfold `upto` 0) THEN SplitOnConclITE
. . |
. . .....falsecase .....
. . 1. ∀d,i,j:ℕ. j-i = d ⇒ no_repeats(ℕ;upto(i;j))
. . 2. i : ℕ
. . 3. j : ℕ
. . 4. ¬i≤j
. . 5. j≤i
. . ⊢  no_repeats(ℕ;nil)  by ObviousConcl
END
```